The Thirty-Third Annual
Eastern Shore High School Mathematics Competition

November 10, 2016

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is **not** permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. If $\log_{10} 3x = \dfrac{1}{4} \log_{10} (x - 12)^4$, then

    a. $x = -6$ b. $x = 3$ c. $x = 6$ d. $x = 3$ or $x = -6$ e. the equation has no real solutions.

2. The sum of the integer-valued solutions of $|3 - 2x - x^2| \leq 3 - 2x - x^2$ is

    a. -5   b. -2   c. -3   d. 2   e. 5

3. Late one Saturday night, Anna, Bill and CeCe are having a grand old time doing arithmetic with the prime numbers. Anna finds the product of the three smallest primes and calls her product A. Bill finds the sum of the next five primes and calls his sum B. CeCe then computes $C = A \times B$. What is the value of C?

    a. 77   b. 97   c. 2010   d. 3,233,230   e. 9,699,690

4. If $\dfrac{4}{w} + \dfrac{4}{x} = \dfrac{4}{y}$ and $wx = y$, then the average (arithmetic mean) of w and x is

    a. $\dfrac{1}{2}$ b. 1   c. 2   d. 4   e. 8

5. Infinitely many circles are constructed so that the radius of the first is 1, the radius of the second is $\dfrac{1}{2}$, the radius of the third is $\dfrac{1}{4}$, and so on (that is, starting with a circle of radius of 1, each subsequent circles radius is half the radius of the previous circle). What is the sum of the areas of these infinitely many circles?

    a. $\dfrac{\pi}{2}$ b. $\dfrac{7\pi}{8}$ c. $\dfrac{8\pi}{7}$ d. $\dfrac{4\pi}{3}$ e. 2π

6. Consider an exponential function of the form $f(x) = ae^{bx}$, where a and b are constants. If $f(2) = e^{-2}$ and $f(3) = e^{-4}$, what is $f(1)$?

    a. e^{-4} b. 1   c. 2   d. e^2 e. e^4

7. A class consists of ten students. The teacher failed to record one student's test grade. The sum of the grades that she had recorded was 698, and the most frequent grade was 76. After recording the tenth score, the class average on a test was 78. What was the unrecorded grade?

    a. 76   b. 77   c. 78   d. 82   e. It cannot be determined.

8. The graph of the cubic $y = x^3 - 9x^2 + 18x + 4$ is symmetric about which point?

   a. (0,4)   b. (1,14)   c. (3,4)   d. (2,12)   e.(4,-4)

9. Statistics for grades on math tests from three different classes are provided in the table below.

	mean	standard deviation
Class 1	79.1	5.7
Class 2	80.6	8.4
Class 3	75.9	15.3

Bobbi is in Class 1, Mary is in Class 2, and Jenny is in Class 3. All three students earned a grade of 81. Who did the best relative to her classmates?

a. There is no difference among the three students.
b. Bobbi did the best relative to her classmates.
c. Mary did the best relative to her classmates.
d. Jenny did the best relative to her classmates.
e. Additional information about each class is needed.

10. In the figure below $\overleftrightarrow{JK} \perp \overleftrightarrow{JL}$ and the circle is tangent to $\overleftrightarrow{JK}$, $\overleftrightarrow{JL}$, and $\overleftrightarrow{KL}$. If d is the length of the diameter of the circle, then
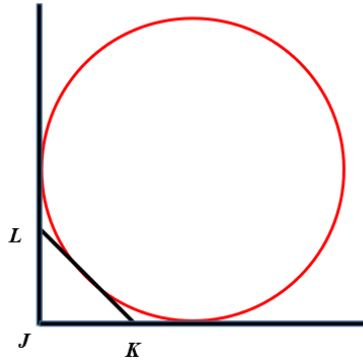
a. $d = (JK)^2$ b. $d = (JL)^2$ c. $d = (JK)^2 + (JL)^2$ d. $d = (JK + KL + JL)$ e. cannot be determined

11. How many distinct factors does 2016 have?

a. 8   b. 12   c. 16   d. 24   e. 36

12. Which one of the following series has the value of 2016?

a. $\displaystyle\sum_{n=0}^{\infty} 2016 \left(\frac{1}{2}\right)^n$

b. $\displaystyle\sum_{n=0}^{\infty} 1512 \left(\frac{1}{4}\right)^n$

c. $\displaystyle\sum_{n=0}^{\infty} 1008 \left(\frac{1}{8}\right)^n$

d. $\displaystyle\sum_{n=0}^{\infty} 1764 \left(\frac{1}{16}\right)^n$

e. $\displaystyle\sum_{n=0}^{\infty} 1890 \left(\frac{1}{32}\right)^n$

13. If $\log_2(x) = c$, then $\log_8(x) + \log_4(x) - \log_{1/2}(x)$ equals

    a. $\dfrac{11}{6}c$ b. $6c$ c. $c^3 + c^2 + c^{-1}$ d. $x^3 + x^2 + x^{-1}$ e. $6x$

14. Each of the 75 people in a room is wearing at least one colored bracelet. The numbers of red bracelets, yellow bracelets, and blue bracelets in the room are all equal. No one is wearing all three colors at once; however, 25 people are wearing a red and a yellow, 20 are wearing a yellow and a blue, and 15 are wearing a red and a blue. There are 10 people wearing only a blue bracelet.

    Which of the following must be true?

    a. Some people are wearing only yellow, but no one is wearing only red.
    b. Some people are wearing only red, but no one is wearing only yellow.
    c. Some people are wearing only red, and some are wearing only yellow.
    d. All of the people who are wearing red are also wearing another color.
    d. None of the above statements is necessarily true.

15. Suppose $\tan(\theta) = -\dfrac{2}{3}$ and $\sin(\theta) > 0$. Then, $\sec(\theta)$ equals

    a. $-\dfrac{\sqrt{13}}{2}$ b. $-\dfrac{\sqrt{13}}{3}$ c. $\dfrac{\sqrt{3}}{2}$ d. $\dfrac{\sqrt{5}}{2}$ e. $\dfrac{\sqrt{13}}{3}$

16. A square is inscribed in a circle (so that the corners of the square all touch the boundary of the circle). If the square has side length 10, what is the area of the circle?

    a. 50π b. 75π c. 80π d. 100π e. 200π

17. For which of the following functions f is $f(.99)$ the greatest?

    a. $f(x) = \sqrt{2016^x}$

    b. $f(x) = x^{2016}$

    c. $f(x) = 2016x$

    d. $f(x) = \dfrac{2016}{x}$

    e. $f(x) = 20^{x^{16}}$

18. Suppose $x_1, x_2, x_3, \ldots$ is an arithmetic sequence such that each term is a whole number, and each term is at least 2 more than the preceding term.

    If the value of x_6 is 40, and some later term in the sequence has value 95, then what is the value of x_1?

    a. 5   b. 15   c. 25   d. 35   e. cannot be determined from this information

19. In the diagram, arc AC is part of a circle, and the segment BC is tangent to this circle. The angle ABC is a right angle. Based on the measurements given, the radius of this circle
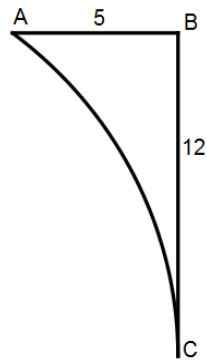
     a. is 13   b. is $\dfrac{169}{10}$ c. is 17   d. is 12π e. cannot be determined.

20. Consider the sequence shown below

$$7, 9, 12, 13, 15, 18, 19, 21, 24, 25, 27, 30, 31, 33, 36, 37, \ldots$$

What is the sum of the first 99 terms of the sequence?

     a. 6436   b. 8218   c. 9306   d. 10395   e. 10428