The Thirty-Fifth Annual
Eastern Shore High School Mathematics Competition

November 8, 2018

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational work on the enclosed form for problems #18, #19 and #20 will then be used as tiebreakers.

1. Sixty-four one inch squares are arranged to form a standard checkerboard pattern. If the squares are white or grey, the radius (R) of the largest circle that can be drawn on the checkerboard that does not touch the interior of a white square is

a. $0 < R \leq \frac{1}{2}$ b. $\frac{1}{2} < R \leq \frac{\sqrt{2}}{2}$ c. $\frac{\sqrt{2}}{2} \leq R \leq \frac{3}{2}$ d. $\frac{3}{2} < R \leq \frac{\sqrt{10}}{2}$ e. Answer not shown

2. Most integers can be represented as the product of prime factors. What is the largest prime factor of 2018000000?

a. 10 b. 19 c. 109 d. 1009 e. 2018

3. If $b > 0$, $\log_b 2 = x$, $\log_b 6 = y$, and $b^z = 3$, then the value of z is

a. $\log_b 3$ b. $\frac{y}{x}$ c. $y - x$ d. $\frac{b^y}{b^x}$ e. Answer not shown

4. Which of the following is equal to $0.\overline{2018}$?

a. $\frac{1009}{4998}$ b. $\frac{2018}{10001}$ c. $\frac{2018}{9999}$ d. $\frac{2018}{9998}$ e. $\frac{201818}{999999}$

5. Let A, B, C, and D be sets of real numbers. Which of the following natural language descriptions describes the set $(A \cup B) \cup (C \cup D)'$?

a. The set of real numbers that are in A or B or not in either C or D.
b. The set of real numbers that are in A or B or the set of those not in C or D.
c. The set of real numbers that are in A or B but not in C or D.
d. The set of real numbers that are in A and B but not in C and D.
e. More than one of these is correct.

6. For the inequality $0.6^{x^2+3x} \geq 1$, what is the largest integer solution?

a. $x = -3$ b. $x = 0$ c. $x = 1$ d. $x = 3$ e. There is no largest integer solution.

7. The sum of the solutions to the equation $(\log_3 x)^2 - 3(\log_3 x) + 2 = 0$ is

a. 3 b. 6 c. 9 d. 12 e. 15

8. Consider the nonzero geometric sequence $2r, 4r^2, 8r^3, \cdots$. The seventh term times twenty-seven is equal to eight times the fourth term. What is the sum of the terms of this sequence?

a. $\frac{1}{3}$ b. 1 c. $\frac{3}{2}$ d. 2 e. 3

9. Let $f(n)$ be the function

$$f(n) = \begin{cases} 1 & n = 1 \\ 2 \cdot f(n/2) + n & n \neq 1 \end{cases}$$

Compute $\big(f(n), f(f(n) - n)\big)$ for $n = 16$.

a. (32, 80)
b. (80, 448)
c. (192, 640)
d. (272, 544)
e. Answer not shown.

10. How many natural numbers less than 2018 are divisible by either 18 or 20, but not both?

a. 190 b. 201 c. 212 d. 223 e. 234

11. Today is Thursday, November 8, 2018. What day of the week will it be exactly 100 years from now, on November 8, 2118? (Hint: during this time frame, every year that is a multiple of 4, *except* for 2100, will be a leap year.)

a. Tuesday
b. Wednesday
c. Thursday
d. Friday
e. Saturday

12. The equation $1 + x + x^2 + x^3 + x^4 + x^5 = \dfrac{1000}{x-1}$ has one positive real solution, t. Which of the following is true?

a. $t < 1$ b. $1 < t < 2$ c. $2 < t < 3$ d. $3 < t < 4$ e. $t > 4$

13. Karla is in a hot air balloon, directly over a point P on the ground. Karla spots a parked car on the ground at an angle of depression of 30°. The balloon rises vertically 50 meters. Now the angle of depression to the car is 60°. How far is it, in meters, from the point P to the car?

a. 25 b. $\dfrac{25\sqrt{3}}{3}$ c. $25\sqrt{3}$ d. $25 + 25\sqrt{3}$ e. 75

14. The scores on a calculus test are 92, 58, 85, 96, 72, 77, 89. Which of the following statistics below would change if the score of 89 was changed to a 93, and the score of 85 was changed to an 81?

a. range b. median c. mean d. mode e. All of these

15. Of thirty students at an Eastern Shore High School, 20 are taking math, 15 are taking English, and 8 are taking both. If one of these students is selected at random, what is the probability that the student is taking neither math nor English?

a. $\dfrac{1}{10}$ b. $\dfrac{4}{15}$ c. $\dfrac{11}{15}$ d. $\dfrac{9}{10}$ e. None of these

16. Suppose a is the 20th term of a non-constant arithmetic sequence, and b is the 18th term. The first term of the sequence is:

a. $\dfrac{19b-17a}{2}$ b. $\sqrt{\dfrac{b^{19}}{a^{17}}}$ c. $\dfrac{19a-17b}{2}$ d. $\dfrac{a-b}{2}$ e. $\sqrt{\dfrac{a}{b}}$

17. A game is played in which a player selects 2 chips from a bag of 5 chips, without replacement. The chips are numbered 1, 3, 3, 4, 5. What is the probability that the player will get a match, i.e., the player selects both 3's?

a. 0 b. $\dfrac{2}{25}$ c. $\dfrac{1}{20}$ d. $\dfrac{1}{10}$ e. $\dfrac{2}{5}$

18. On the recently discovered (very small) planet *Nova* in the *Sol* system (near Betelgeuse), without exception the 63 inhabitants were found to speak three distinct languages: *Yod (Y), Aleph(A), and Mem(M).* Upon tallying the number of inhabitants who can speak at least one of A, M, or Y, the exact total is 100 – so clearly some Novarians can speak more than one language. If there are 19 Y/M-bilingual Novarians, how many A-speaking Novarians can speak at least one other language?

a. 0 b. 2^3 c. 18 d. 21 e. 37

19. The arithmetic sequence $a_1, a_2, a_3, \cdots$ has 29 as its tenth term and 62 as its twenty-first term. The arithmetic sequence $b_1, b_2, b_3, \cdots$ has 35 as its eighth term and 103 as its twenty-fifth term. Find the 999th term of the sequence:

$$a_1, b_1, a_1+b_1, a_2, b_2, a_2+b_2, a_3, b_3, a_3+b_3, \cdots$$

a. 1001 b.1335 c. 2333 d. 2999 e. 6995

20. Consider the twenty lattice points in a 3 by 4 rectangular grid. If all possible squares are formed by connecting four of the 20 lattice points, what is the total area (T) of those squares? Given: the vertices of the 3 by 4 rectangle are 4 of the 20 lattice points; the other 16 lattice points are either on the sides of the 3 by 4 rectangle or in the interior of the 3 by 4 rectangle.

a. $0 < T \leq 12$ b. $12 < T \leq 28$ c. $28 < T \leq 36$ d. $36 < T \leq 54$ e. Answer not shown