The Thirty-Fifth Annual
Eastern Shore High School Mathematics Competition

November 8, 2018

Team Contest Exam

Instructions

Answer as many questions as possible in the time provided. To receive full credit for a correct solution, show all work and provide a clearly written explanation. Solutions will be judged based on correctness, completeness and clarity. (Little credit, if any, will be given for a solution consisting of just a number or a single sentence.) Calculators are allowed **only** on the team contest exam.

All work and answers must be written on the provided sheets of plain white paper. Use only one side of each sheet of paper, and start each new problem on a new sheet of paper. Write the name of the school which you are representing at the top of each sheet that you turn in for scoring.

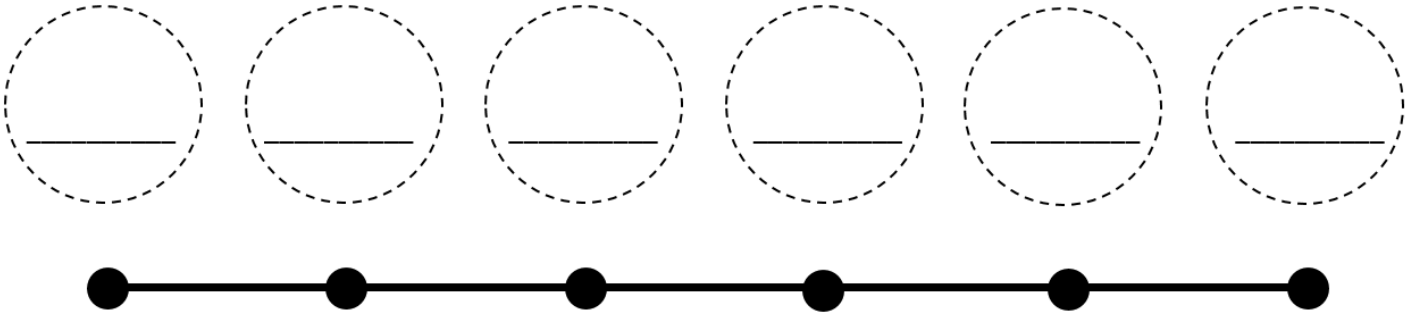
At the start of the team round, your team will receive a copy of only Problem 1. Your team must submit a response to Problem 1 within the first 15 minutes of the team round time interval.

When you submit your response for Problem 1, you will receive a copy of Problem 2 and a copy of Problem 3. Your team will then have the time remaining in the team round to complete a response for each problem.

Note: if your team completes Problem 1 before the end of the allotted time, you may submit it and receive copies of Problem 2 and Problem 3 in advance.

1. Six students were discussing how to color the six vertices on the graph shown below. They decided they would use blue, green, orange, pink, red and yellow to color each vertex a different color. Student A said she doesn't want the yellow vertex connected to the green vertex or the blue vertex connected to the orange vertex, or the red vertex connected to the pink vertex. Student B doesn't want the pink vertex connected to the yellow vertex or the green vertex connected to the blue vertex. Student C insists that the orange vertex cannot be connected to the yellow vertex, the blue vertex cannot be connected to the pink vertex and the green vertex cannot be connected to the red vertex. Student D refuses to let the yellow vertex be connected to the red vertex or the green vertex be connected to the orange vertex. Arrange the colors on the "graph" shown below to satisfy all of the students' conditions.

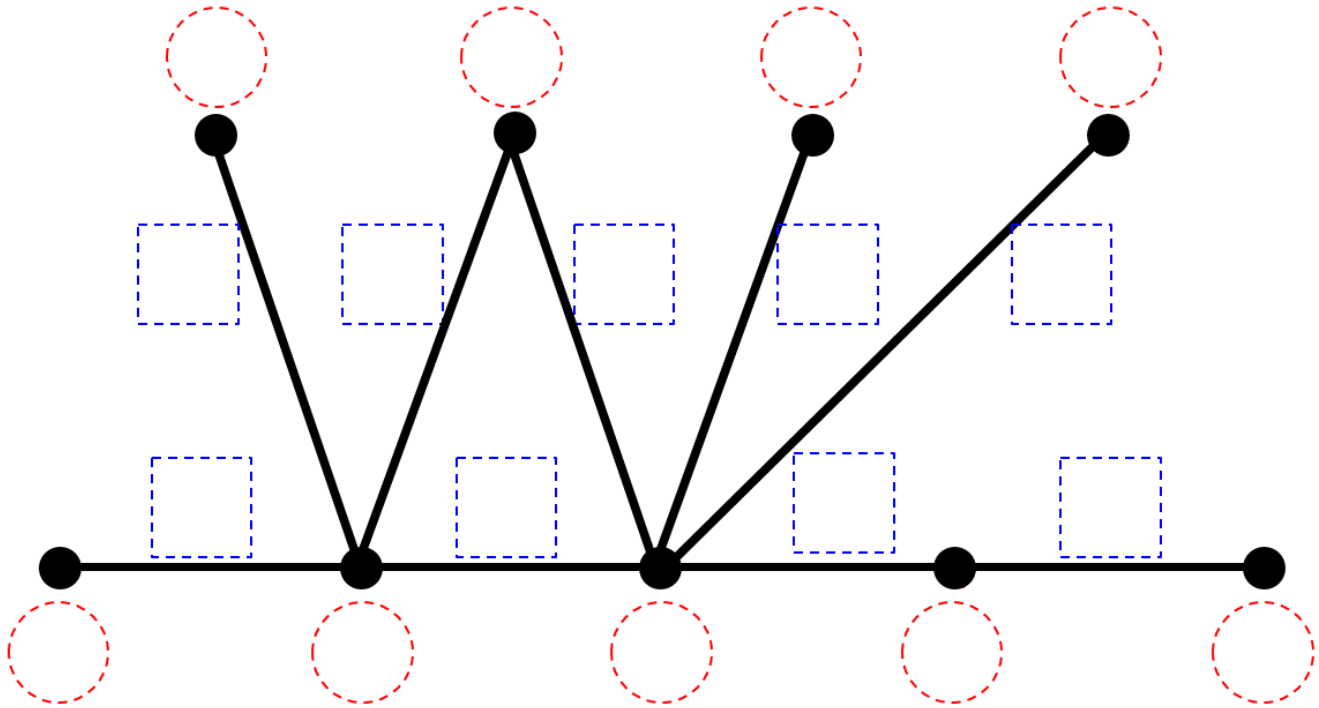
   Enclosed in your team packet you should find a set of "six color coded vertices." If you do not have the set in your packet, please notify a proctor.



2. Arrange the "vertices" (numbered 0-8) and the "segments" (numbered 4-12) on the graph shown below so that the **sum** of the numbers on any two vertices connected by a segment is equal to the number on the connecting segment.

   Enclosed in your team packet you should find a set of "vertices numbered 0-8" and a set of "segments numbered 4-12." If you do not have both sets in your packet, please notify a proctor.

Solution:



3. The image below portrays the first few steps of the following process:

   - Draw a circle of radius r, and color its interior black.
   - Inscribe an equilateral triangle in the circle, as shown. Color the triangle's interior white.
   - Inscribe a circle in the resulting equilateral triangle. Color the circle's interior black.
   - Inscribe an equilateral triangle in this new circle, as shown. Color the triangle's interior white.
   - Inscribe a circle in the resulting equilateral triangle. Color the circle's interior black.
   - ...

   Suppose this process is continued indefinitely, and a dart is thrown at the resulting region. Assume that the dart will land somewhere within that region bounded by the initial circle of radius r. What is the probability that the dart lands in the black portion of the region? Provide justification for your answer.