

# DEVELOPING STUDENTS' CONCEPTUAL UNDERSTANDING OF PLACE VALUE AND DECIMALS

Jill Hitchens and Jessica Williams  
Faculty Mentor: Dr. Claudia Burgess

# Introduction

# Common Misconceptions

- ⦿ Apply knowledge of whole numbers to decimals. (Durkin & Rittle-Johnson, 2014).
  - Ex. students may believe that 0.45 is greater than 0.8 because 45 is greater than 8
- ⦿ Believe if there is a zero at the end of a decimal number, that the value increases (Durkin & Rittle-Johnson, 2014)
  - Ex. students might believe that 0.89 has a lesser value than 0.8900
- ⦿ Students are often told to “line up the decimal point” (Ashlock, 2010).
  - Ex. believing that  $0.7 + 0.7 = 0.14$

# Purpose

The purpose of this study was to examine students' thinking about whole number place value and the base ten system and how their understandings of these topics influenced their learning of decimals and decimal computations.

# Research Question

How can students' proficiency be improved in the areas of whole number place value, decimal place value, and decimal computation?

# Theoretical Framework

# Five Strands of Mathematical Proficiency

## Conceptual Understanding

Comprehension of mathematical concepts, operations, and relations

## Procedural Fluency

Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

## Strategic Competence

Ability to formulate, represent, and solve mathematical problems

## Adaptive Reasoning

Capacity for logical thought, reflection, explanation, and justification

## Productive Disposition

Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

# Additional Educational Research

- ◎ Build upon students' prior knowledge (Carpenter et al., 2015)
- ◎ Build upon students' knowledge of place value to develop their understanding of decimals (Ashlock, 2010)
- ◎ Use visual models to illustrate the conceptual structure of decimals (National Governors Association & Council of Chief State School Officers, 2010)
- ◎ Use inquiry-oriented instruction (Reys et al., 2014).



# Methodology:

Participants and Procedures

Data Gathering and Analysis

# Learning Trajectory for Place Value and Decimals

4.NBT.1

- Recognize a digit in one place represents ten times what it represents in the place to its right

5.NBT.3.a

- Read and write decimals to thousandths using expanded notation and place value.

5.NBT.3.b

- Compare decimals to thousandths using values of digits

5.NBT.1

- Recognize a digit in one place represents ten times what it represents in the place to the right and  $\frac{1}{10}$ th of what it represents to the left.

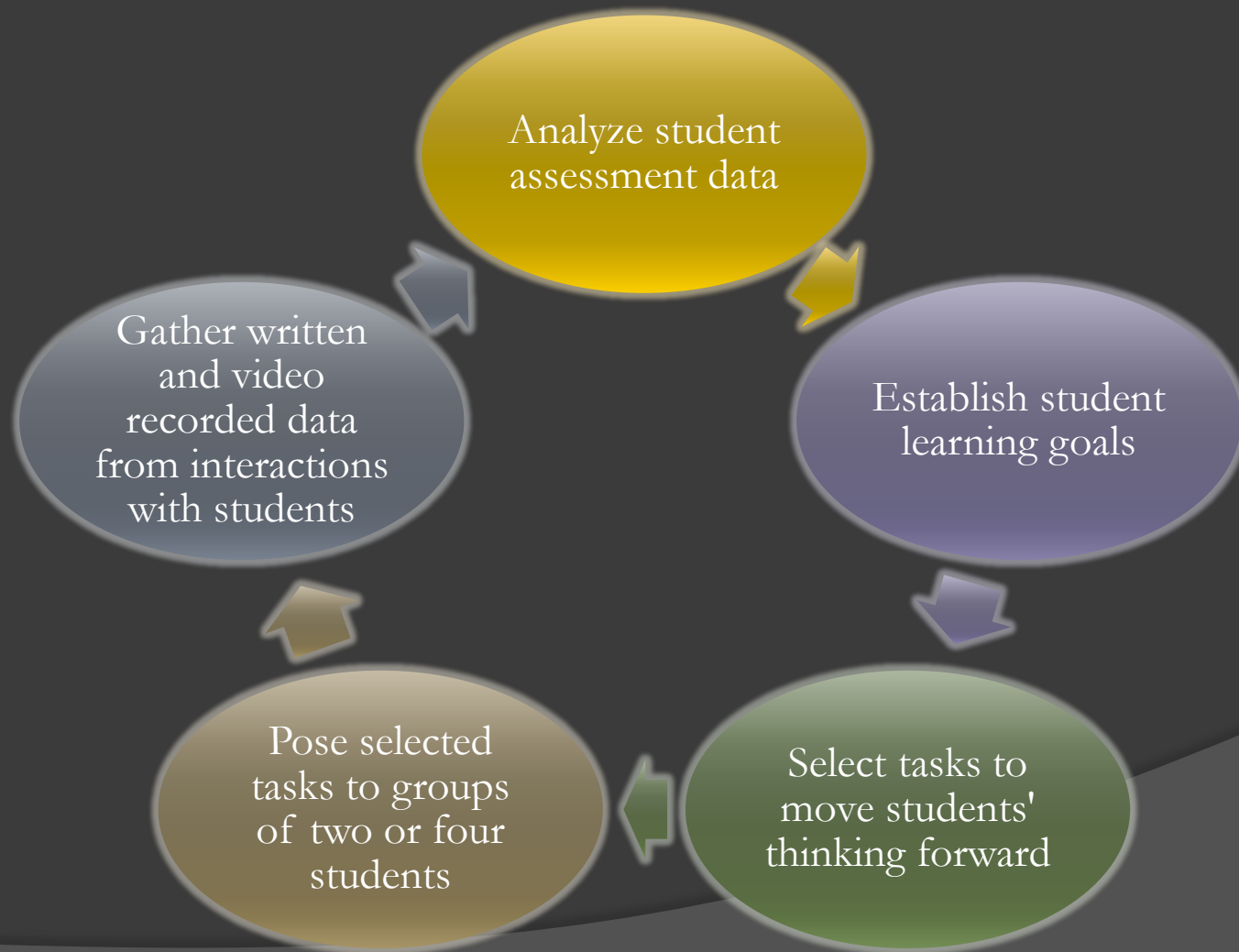
5.NBT.7

- Add, subtract, multiply, and divide decimals to hundredths and explain.

# Participants

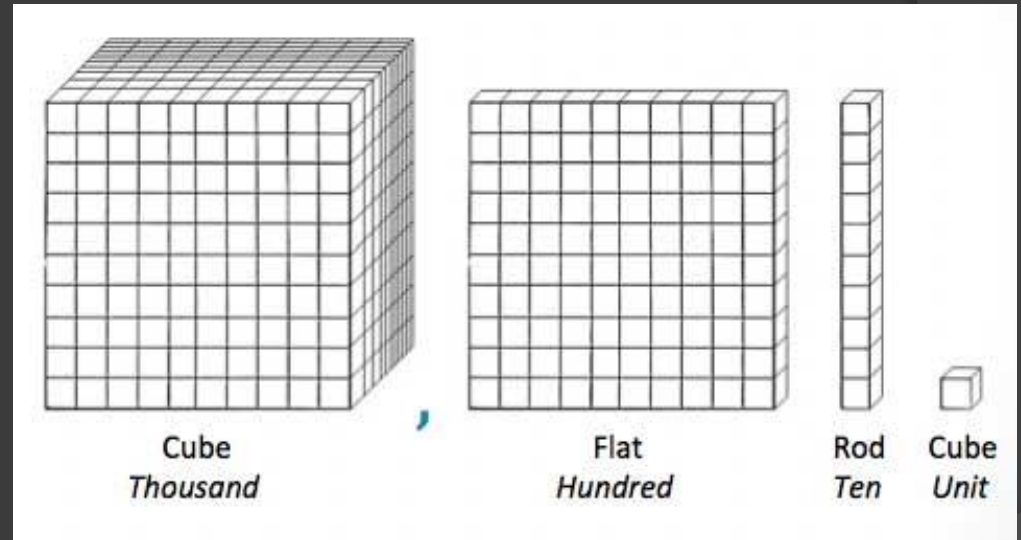
- ◎ 4 students entering fifth grade in fall 2015
- ◎ Pseudonyms:
  - Alex, Bethany, Christina, Daniel
- ◎ Participated in:
  - 30-minute initial interview
  - Seven 1-hour tutoring sessions
  - 30-minute post interview
- ◎ Participation Rate: 100%

# PATHWAYS Cycle of Integrated Teaching and Research



# Sample Interview Questions

1. What would be the value of the next block to the left [of the thousands cube]?
2. Add  $2.4 + 10.03$
3. Subtract  $12.0 - 0.145$
4. Multiply  $0.2 \times 3$



# Initial Assessment Results

- Students had a wide range of abilities
- Demonstrated procedural fluency without conceptual understanding

Add  $2.4 + 10.03$

$$\begin{array}{r} 2.4 \\ + 1003 \\ \hline 1027 \end{array}$$

$$\begin{array}{r} + 10.03 \\ 2.4 \\ \hline 12.43 \end{array}$$

# Initial Assessment Results

6. What would be the value of the next block to the left? What would its value be? How you you write it?  
(4.NBT.2 ~~2-014(0)11-1~~)

2,000    1,000,000    100,000    2,000

Christina

Bethany

Alex

Daniel

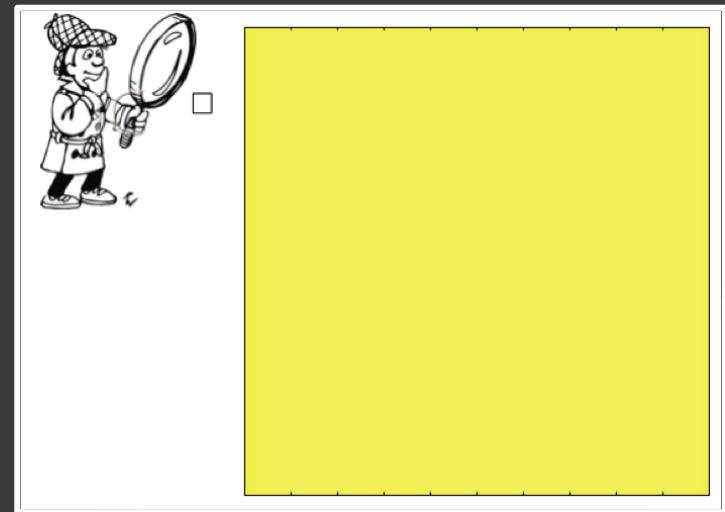
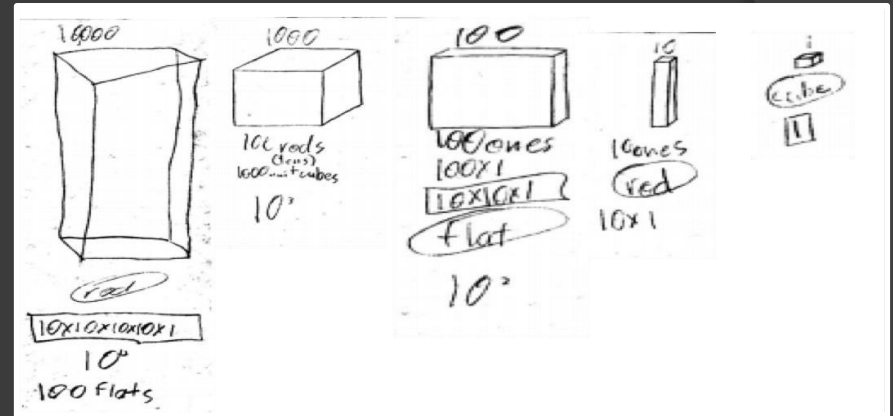
Write 347.392 in expanded form

300.000 + 40.000 + 7.000 + .300 + .090 + .002

347.392

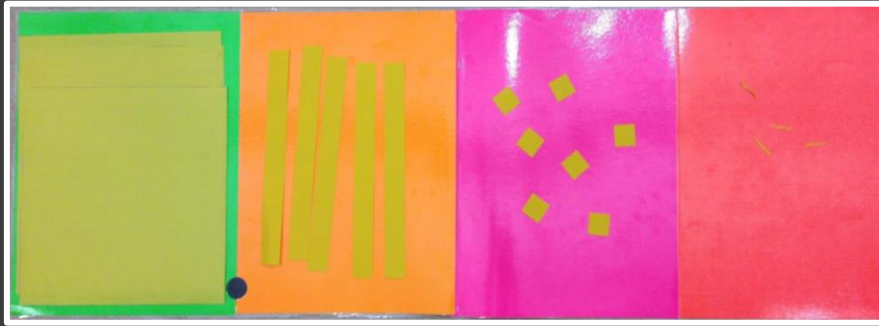
# Instructional Cluster 1

- Understanding whole number and decimal place value
- Use manipulatives to represent various numbers
- Discover patterns in base ten system

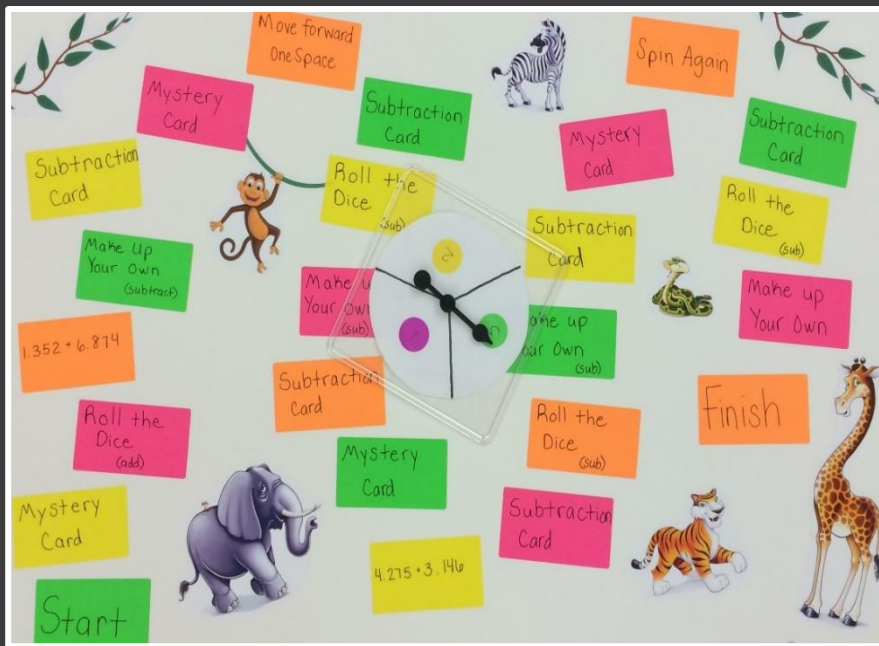




# Instructional Cluster 2



- Used manipulatives
- Adding and subtracting decimals
- Concepts of regrouping
- Differentiated instruction
- Small group work



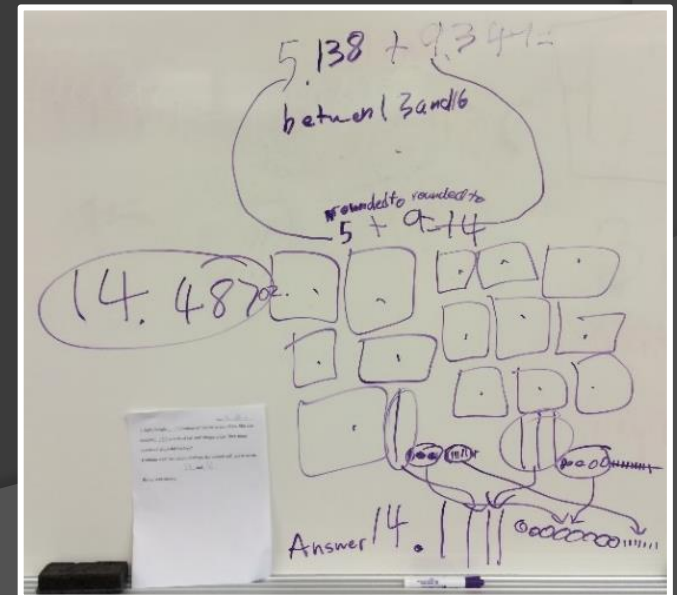
# Instructional Cluster 3

- Multiplication of a whole number by a decimal, ex.  $2 \times 0.3$
- Open-ended word problems
- Represented numbers with drawings
- Explained reasoning

1. Sally bought 5.138 ounces of regular potato chips. She also bought 4.349 ounces of salt and vinegar chips. How many ounces of chips did she buy?

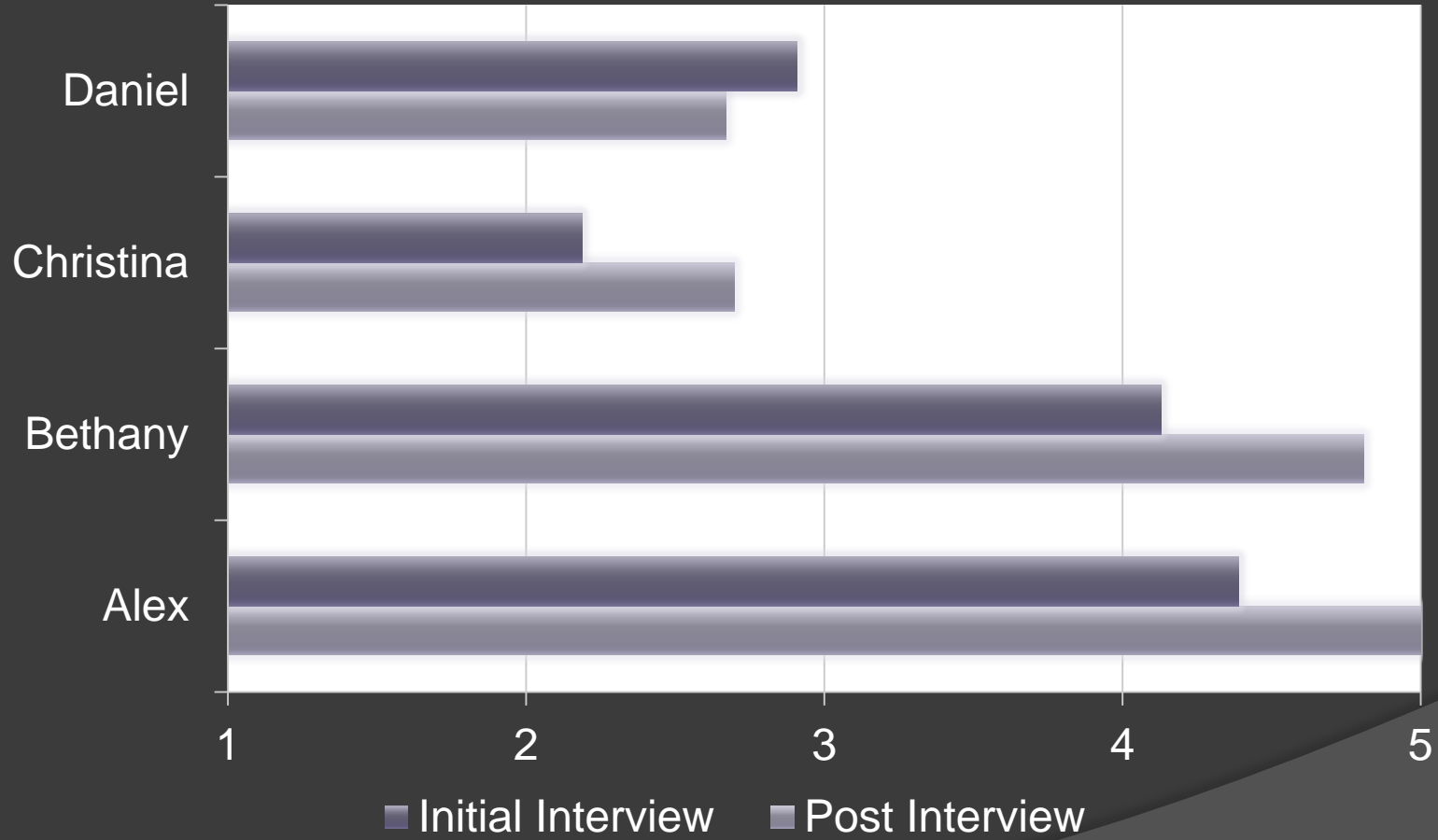
Estimate what two whole numbers the answer will fall between.

13 and 16



# Post Assessment Results

# Initial and Post Interview Results



# Results

Alex

Multiply  $2.5 \times 0.5$  (Can you use a picture to justify your answer?)

1.25

Christina and Daniel were able to accurately read the decimals on the post interview.

Bethany

∴ In expanded form

$$300.000 + 40.000 + 7.000 + .300 + .090 + .002 =$$

# Reflection and Discussion

- ⦿ An understanding of whole numbers is essential
  - Initial assessment
- ⦿ Do not move through standards too quickly
  - Multiple experiences with each concept
- ⦿ Assessment should match instruction method
  - Manipulatives

# References

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