Developing Students' Proficiency in 7th Grade Common Core Statistics

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Introduction

Background:
Research indicates that students in elementary and middle school often have difficulty understanding statistics and probability. In particular, students generally struggle with the language of probability, both simple and compound events, and making connections/differentiations between experimental and theoretical probability (Watson, 2005).

Purpose:
The purpose of this study is to examine how students think about and approach probability and statistics prescribed in the 7th Grade Common Core State Standards.

Research Question:
How can students' mathematical proficiency be developed for the language of probability, theoretical and experimental probability, and simple and compound events?

References:
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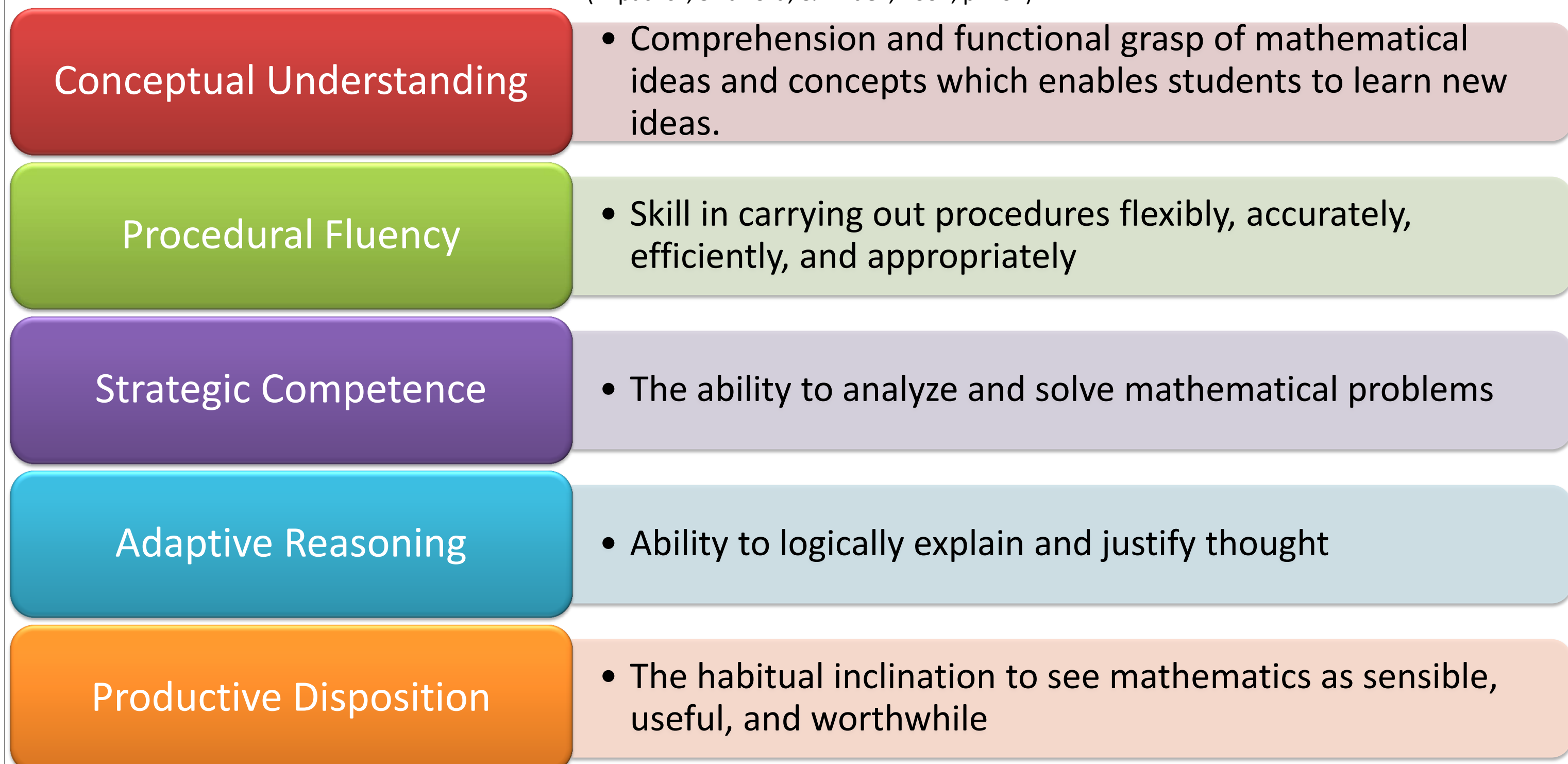Romberg, T. et al. (2003). Mathematics in context: Take a chance. Chicago: Britannica.

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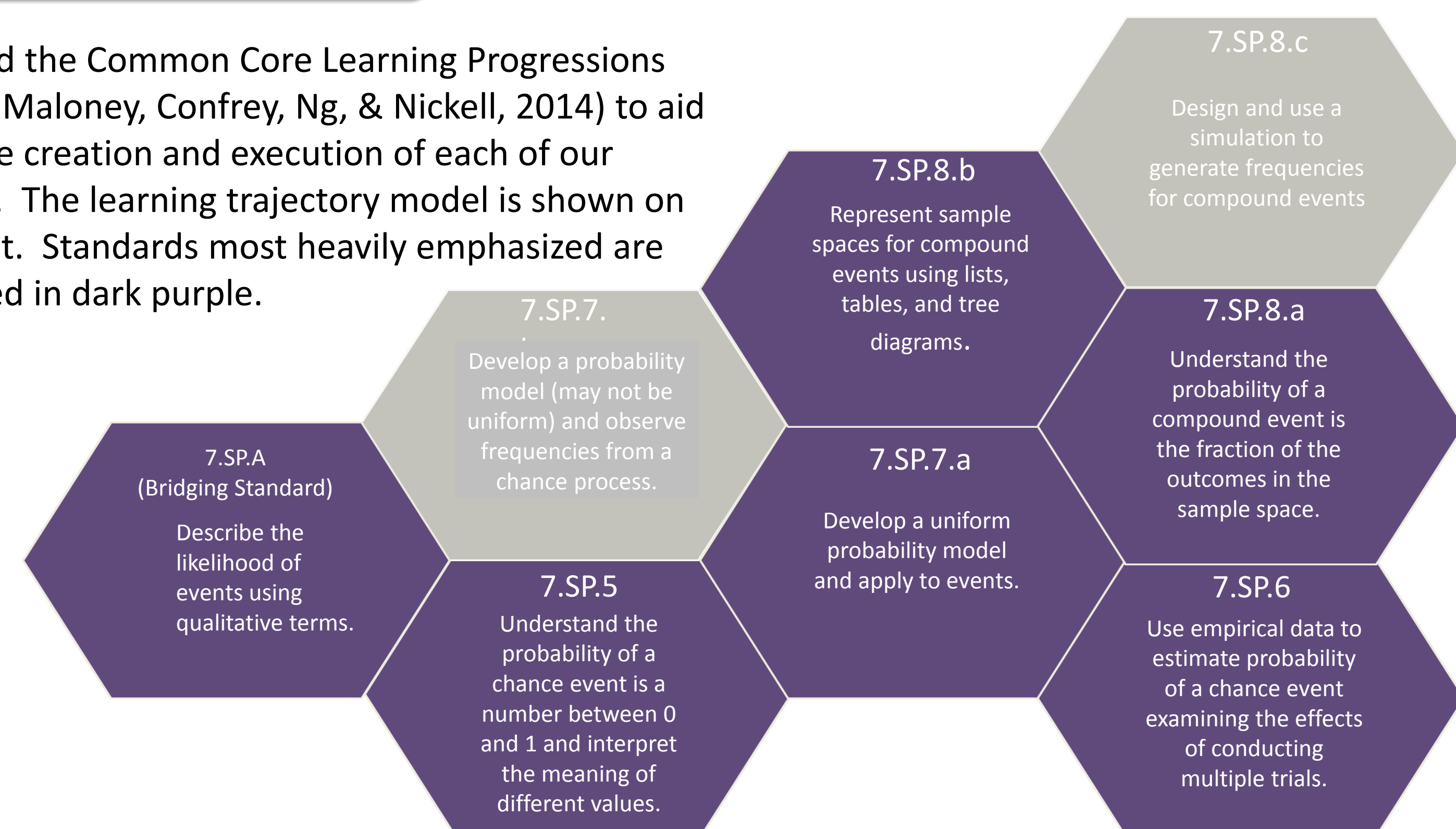
Theoretical framework

Five Strands of Mathematical Proficiency
(Kilpatrick, Swafford, & Findell, 2001, p. 164)

- **Conceptual Understanding** • Comprehension and functional grasp of mathematical ideas and concepts which enables students to learn new ideas.
- **Procedural Fluency** • Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic Competence** • The ability to analyze and solve mathematical problems
- **Adaptive Reasoning** • Ability to logically explain and justify thought
- **Productive Disposition** • The habitual inclination to see mathematics as sensible, useful, and worthwhile

We used the Common Core Learning Progressions Model (Maloney, Confrey, Ng, & Nickell, 2014) to aid us in the creation and execution of each of our lessons. The learning trajectory model is shown on the right. Standards most heavily emphasized are indicated in dark purple.
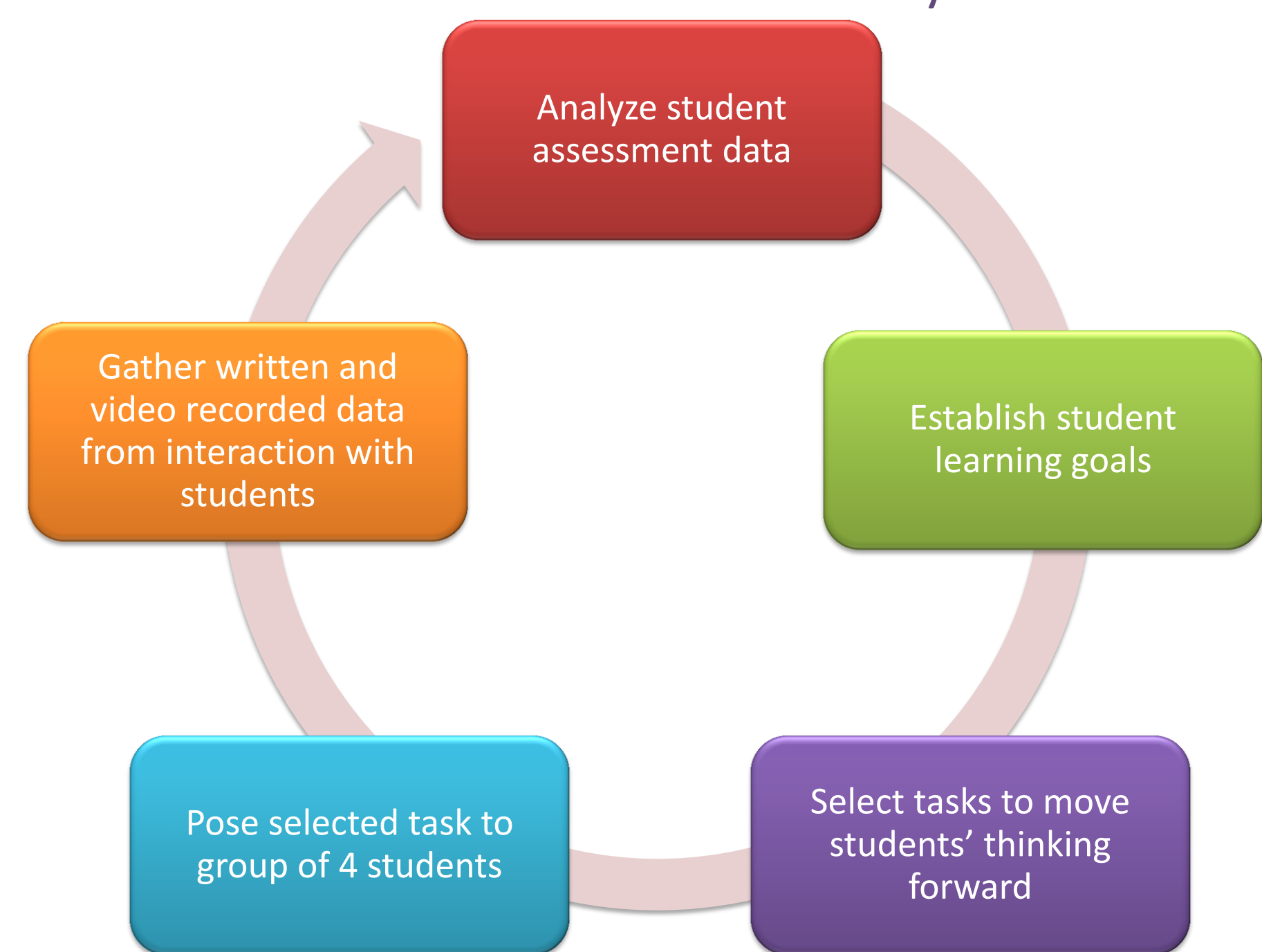

Learning trajectory hexagon diagram:
- 7.SP.A (Bridging Standard) — Describe the likelihood of events using qualitative terms.
- 7.SP.5 — Understand the probability of a chance event is a number between 0 and 1 and interpret the meaning of different values.
- 7.SP.7. — Develop a probability model (may not be uniform) and observe frequencies from a chance process.
- 7.SP.7.a — Develop a uniform probability model and apply it to events.
- 7.SP.6 — Use empirical data to estimate probability of a chance event examining the effects of conducting multiple trials.
- 7.SP.8.a — Understand the probability of a compound event is the fraction of the outcomes in the sample space.
- 7.SP.8.b — Represent sample spaces for compound events using lists, tables, and tree diagrams.
- 7.SP.8.c — Design and use a simulation to generate frequencies for compound events

Methodology – Participants and procedure

Participants:
Four students, two male and two female, moving from 6th grade into 7th. The students participated in seven one hour sessions and two interviews (one student missed one of the one hour sessions). Each student has been assigned a fictitious name and will be referred to with the pseudonyms: Thomas, Nathan, Natalia, and Katherine.

PATHWAYS Instructional Cycle

- Analyze student assessment data
- Establish student learning goals
- Select tasks to move students' thinking forward
- Pose selected task to group of 4 students
- Gather written and video recorded data from interaction with students

Methodology – Data gathering and analysis

Interview Script:
The interview script for both the pre- and post-assessments was used to conduct 30-minute clinical interviews with students. All questions were aligned with our targeted Common Core State Standards and sequenced according to the learning progression. We asked students to explain their answers and to think aloud as they answered the questions.

SAMPLE INTERVIEW QUESTIONS

Sample Question 1
The two fair spinners shown above are part of a carnival game. A player wins a prize only when both arrows land on black after each has been spun once.
James thinks he has a 50-50 chance of winning. Do you agree?
(Zawojewski & Shaughnessy, 2000, p. 263)

Sample Question 2
Choose words from the word bank to describe how likely each event is. Explain your thinking for each item.

Word Bank	
Certain	Possible
Likely	Impossible
Unlikely	Rare
Evenly Likely	Almost Likely

1. You will have a test in math sometime this year.
2. It will rain in your town sometime in the next month.
3. You will meet the President of the United States sometime during your life.
4. You will roll a "7" on a normal number cube.
5. In a room of 367 people, two people will have the same birthday.
(Romberg et al., 2003, p. 11)

Sample Question 3
Match 'Em Up (Romberg et al., 2003, p. 13)
Dan is doing an experiment. He is holding pieces of paper of equal size, numbered 1-20. He is going to pick a number from the bag. Here are some of the possible outcomes for the number he will pick:
a. It will be even.
b. It will be divisible by five.
c. It will be a 1 or a 2.
d. The digits in the number will add up to 12.
e. It will be smaller than 16.
*Put the five statements on a ladder like the one to the right and explain why you put them where you did.

Sample Question 4
What is the probability of rolling a sum of 7 with two fair dice? What is the probability of rolling a 2 with two fair dice? Explain how you know.

Each of the hour long sessions with the students was video recorded and then transcribed. The transcriptions were then analyzed and coded based on the 5 Strands of Mathematical Proficiency. Each week the students' strengths and weaknesses for the various strands were identified and recorded. Finally, each week we tracked the students' progress along the learning progression.

Empirical Teaching and Learning Trajectory:

Initial Assessment Results (Week 1)
During the initial interview the students were asked a variety of statistical and analysis questions. Each of the four students generally seemed to struggle with the language of probability, theoretical vs. experimental probability, as well as how to determine the probability of a compound event.

Language of Probability
The students were asked to pick the best qualitative term (e.g. certain, likely, impossible, etc.) from a word bank to describe a given scenario. For example, when the students were asked to determine the probability of the event "it will rain in your town sometime in the next month" the students' answers varied from certain, to likely, to rare. However, when students were asked to determine the probability of the event "you will roll a '7' on a normal number cube" each of the students could determine that the probability for this event is impossible.
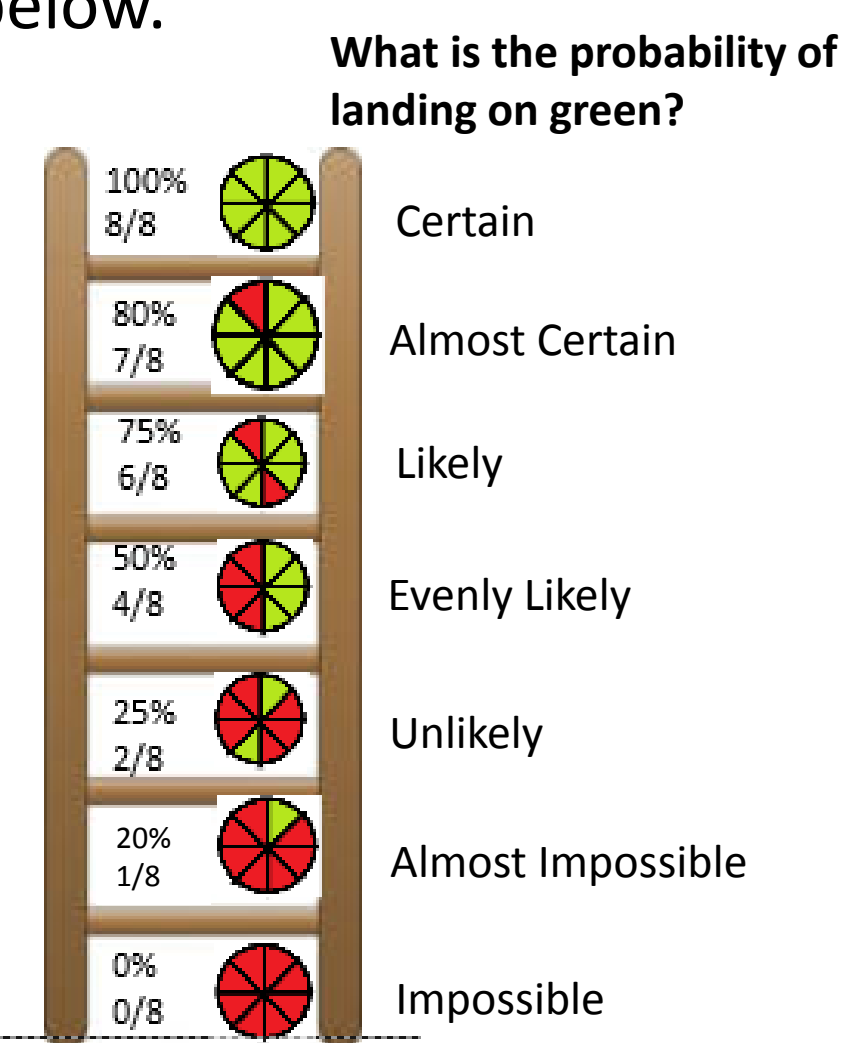
Theoretical vs. Experimental Probability
Three of the four students could accurately determine that when a coin is tossed 100 times it is not guaranteed the results will be exactly 50 heads and 50 tails. However, when the students were asked to determine how the results of a coin flip experiment would change with more trials, none of the students' explanations demonstrated that they had a completely accurate understanding of long term experimental probability.

Simple and Compound Events
Each of the four students could accurately determine theoretical probabilities for simple events (e.g. the probability of rolling a 3 on a single fair die is 1/6). However, none of the students could determine accurate theoretical probabilities for a compound event (e.g. sample question one, shown above).

Qualitative Terms (Week 2, 3)
These lessons focused on having students describe probabilities using qualitative terms. In the initial lessons, students were asked to describe various events using terms such as certain, likely, unlikely, and impossible. As a result of our classroom discourse, students were able to create a probability ladder on which they correctly sequenced qualitative probability terms and corresponding quantitative probabilities for a simple experiment with a spinner, as shown below.

What is the probability of landing on green?
- 100% 8/8 — Certain
- 80% 7/8 — Almost Certain
- 75% 6/8 — Likely
- 50% 4/8 — Evenly Likely
- 25% 2/8 — Unlikely
- 20% 1/8 — Almost Impossible
- 0% 0/8 — Impossible

Theoretical and Experimental Probabilities (Week 4, 5)
These lessons focused on having students identify and describe differences between theoretical and experimental probability. During week 5's lesson students discussed a TinkerPlots file showing that experimental probability is more likely to mirror the theoretical probability after more trials have been completed.

The TinkerPlots simulation on the left shows the results of rolling a single die 1,000 times.
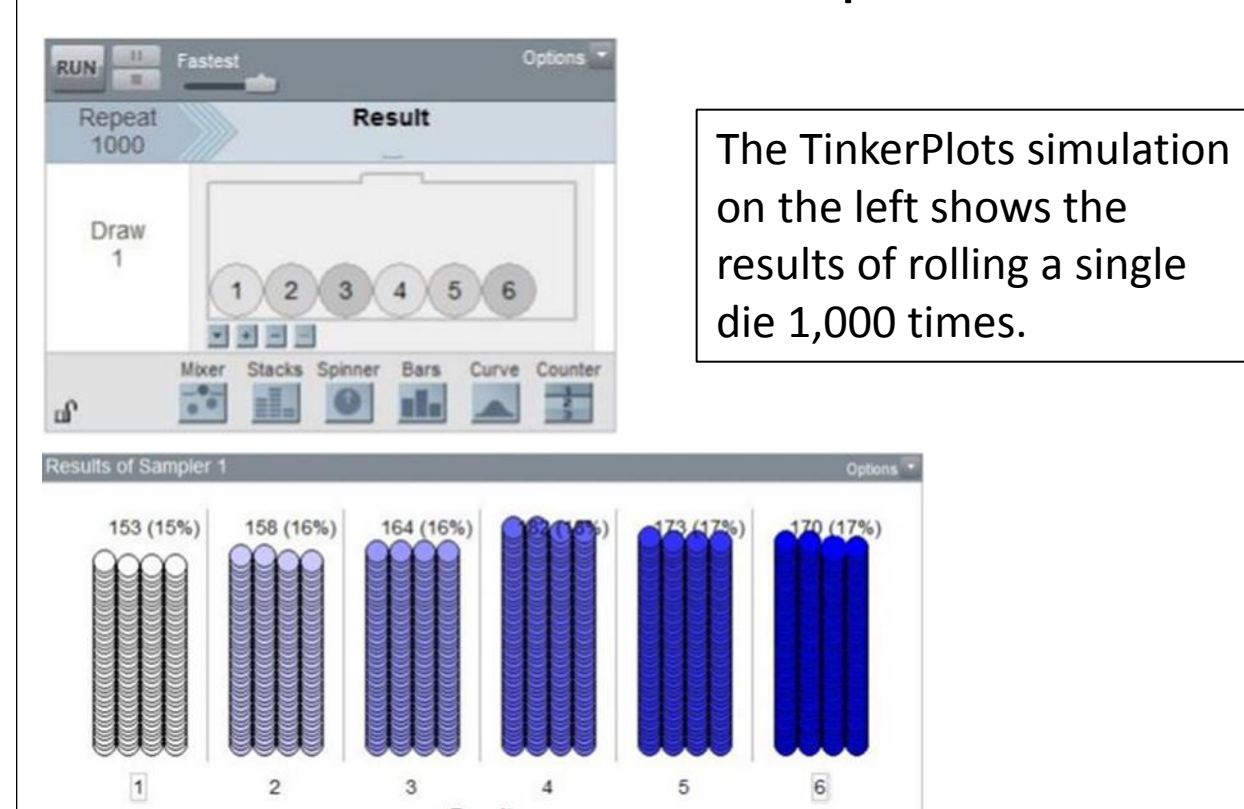
After the conclusion of these lessons students were able to accurately identify scenarios as theoretical or experimental probabilities.

Sample Scenario:
Malik looks at a spinner that is divided into 6 equal sections; red, yellow, blue, green, purple, and orange. He says that the probability of landing on the green section is 1/6

Compound Events (week 7 & 8)
These lessons focused on having students analyze compound events and create probabilities for the events. In both lessons students were prompted to use probability models to assist them in determining theoretical probability.

Penny	Nickel	Results
Heads	Heads	HH
	Tails	HT
Tails	Heads	TH
	Tails	TT
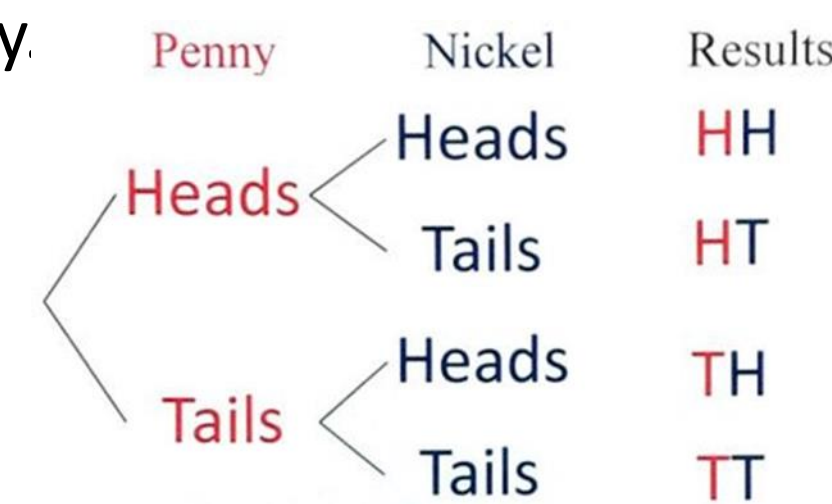
After completing these lessons students were able to complete both tree diagrams and tables for multiple compound events. However most students still struggled to accurately determine the theoretical probability of compound events.

Post Assessment Results (Week 9)
During the post assessment interview the students were asked the same statistical questions that they answered during the pre assessment.

Language of Probability
The students were asked to pick the best qualitative term (e.g. certain, likely, impossible, etc.) from a word bank to describe a given scenario. When the students were now asked to determine the probability for the question "it will rain in your town sometime in the next month", three of the four students were now able to reason that this event is likely to happen. In addition, when students were asked to determine the probability of the event "you will roll a '7' on a normal number cube" each of the students could still accurately determine that the probability for this event is impossible.

Theoretical vs. Experimental Probability
Four out of four students could now accurately determine that when a coin is tossed 100 times that it is not guaranteed that the results will be exactly 50 heads and 50 tails. Additionally, when the students were again asked to determine how the results of a coin flip experiment would change with more trials, all of the students' responses were similar to their initial responses with little evidence showing their understanding of long term experimental probability.

Simple and Compound Events
Each of the four students could still accurately determine theoretical probabilities for simple events (e.g. the probability of rolling a 3 on a single fair die is 1/6). Furthermore, two of the four students could now construct a table/diagrams to help them determine the probability for a compound event. Of those two students, one was able to arrive at the correct answer for sample question one, shown above.

Reflection and discussion: As we concluded our research it was apparent that some CCSSM Standards were more difficult for students to attain than others. Students struggled the most with standard 7.SP.A and standard 7.SP.8.B. For standard 7.SP.A, it was initially challenging for students to use qualitative terms when describing probabilistic events. This indicates the importance of taking time to allow students to gain deep understanding of these terms even though they are included in a "bridging standard" not explicitly written into the CCSSM. The problems that students faced in relation to standard 7.SP.8.B stemmed from the fact that they struggled to understand when tables and tree diagrams should be used to determine probabilities. We believe this occurred because students thought they could always see the sample space at a glance. When students were asked to determine probabilities for compound events (rather than simple events) they failed to understand how drastically the sample space was altered. For example, none of the students had difficulty determining the probability of rolling a three on a single fair die; however, when they were asked to determine the probability of rolling a sum of 7 with two fair die, each of the students gave a fraction response with 12 as the denominator. Their reasoning for this was that they could see that each die has 6 sides and that six plus six equals twelve possible outcomes. Based on these observations we suggest that when teaching statistics and probability, teachers should encourage students to first list out the sample space prior to determining any theoretical probability. We propose this method because if students were in the habit of first listing out the sample space, even for simple events, they may naturally start trying to systematically diagram the sample spaces for compound events as well. In addition, to help students further develop a sense of sample space with compound events, we suggest that students more gradually shift from using their own methods to using more formal methods (i.e. lists, tables, and diagrams) for determining compound probabilities.