Developing 5th Graders' Understanding of Fraction Multiplication

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Introduction

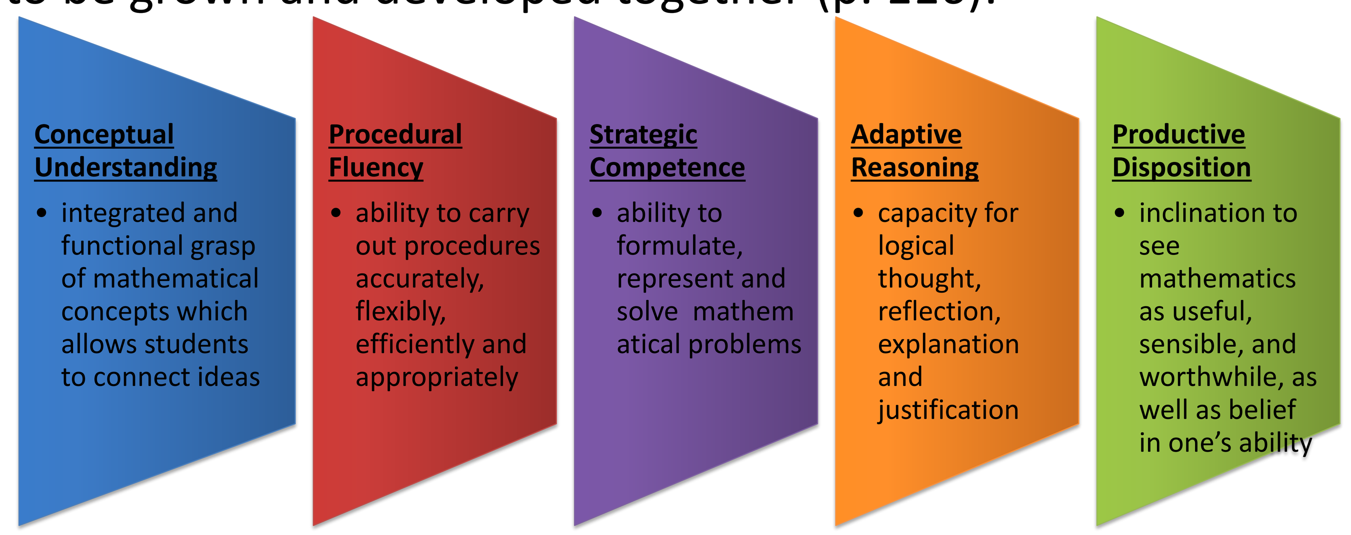
Wilson et al. (2011) stated "It is well documented that even in middle school students continue to struggle with concepts of rational numbers" (p.231). In particular, students show difficulty in fraction multiplication. Although the algorithm for multiplying fractions is much easier to learn than many others, Tsankova et al. (2010) stated "it has been found that students often have difficulties applying the algorithm with flexibility" (p.281). To encourage growth in understanding, visual models were emphasized in this study to provide students with the opportunity to identify fraction multiplication as a conceptual relationship between rational numbers (Webel et al., 2016)

Purpose: We aimed to design methods to build students' conceptual understanding of fraction multiplication.

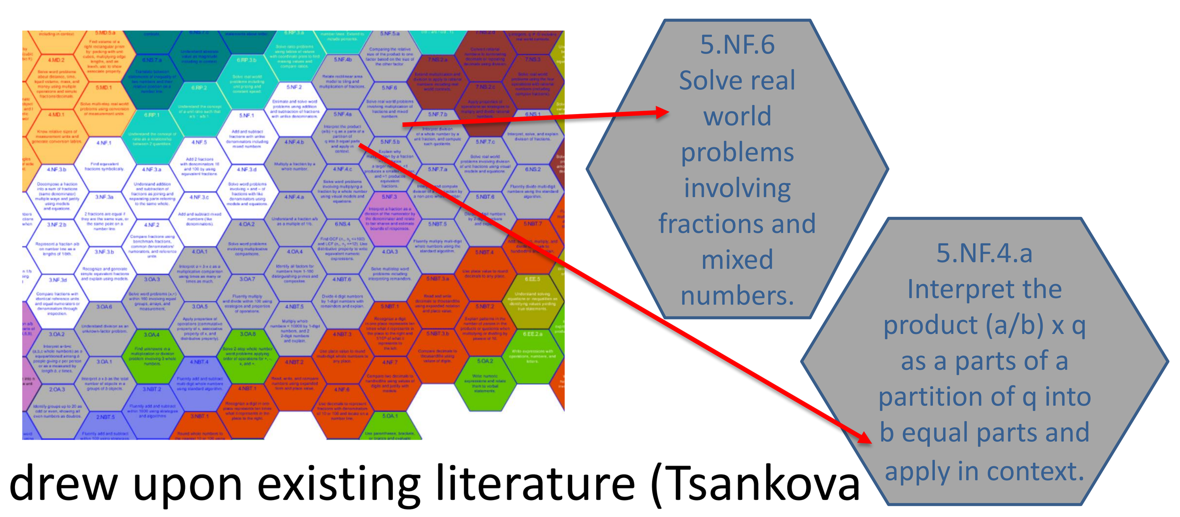
Research Question: What are the characteristics of an instructional sequence that helps students understand fraction multiplication?

Theoretical framework

Kilpatrick et al. (2001) offered the following definitions for the 5 Strands of Mathematical Proficiency which are meant to be grown and developed together (p. 116):



We focused on two specific parts of a learning trajectory (Confrey et al., 2012) for fraction multiplication:



We drew upon existing literature (Tsankova et al., 2010; Webel et al., 2016; Wilson et al., 2011) to select and design robust visual models and accompanying problems for our instructional sequence.

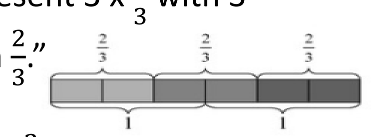
Methodology

We worked with four students, who all just finished the 5th grade, over seven one-hour periods and two 30-minute interviews per student. Two students attended all sessions. One student missed two of the teaching sessions and another student missed one. During the pre-interviews there were some problems that revealed the extremes of our students' thinking.

Key problems were as follows:

Key problem one (to the right) dealt with students' knowledge of repeated addition and fractional parts. They were to use pictures and diagrams To justify their reasoning.

Makayla said, "I can represent 3 x $\frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$."



Connor said, " I know that $\frac{2}{3}$ x 3 can be thought of as $\frac{2}{3}$ of 3. Is 3 copies of $\frac{2}{3}$ the same as $\frac{2}{3}$ of 3?"

a. Draw a diagram to represent $\frac{2}{3}$ of 3.
b. Explain why your picture and Makayla's picture together show that 3 x $\frac{2}{3}$ = $\frac{2}{3}$ x 3.

(Illustrative Mathematics a, n.d.)

The members of a cross country team like to continue their training on their own during the summer. Nero ran 1 ½ miles one day.

a. Lily ran 3 times as far as Nero. How far did Lily run?
b. Jorge ran ¾ times as far a Nero. How far did Jorge run?
c. Belinda ran 2 1/3 times as far as Nero. How far did Belinda run?

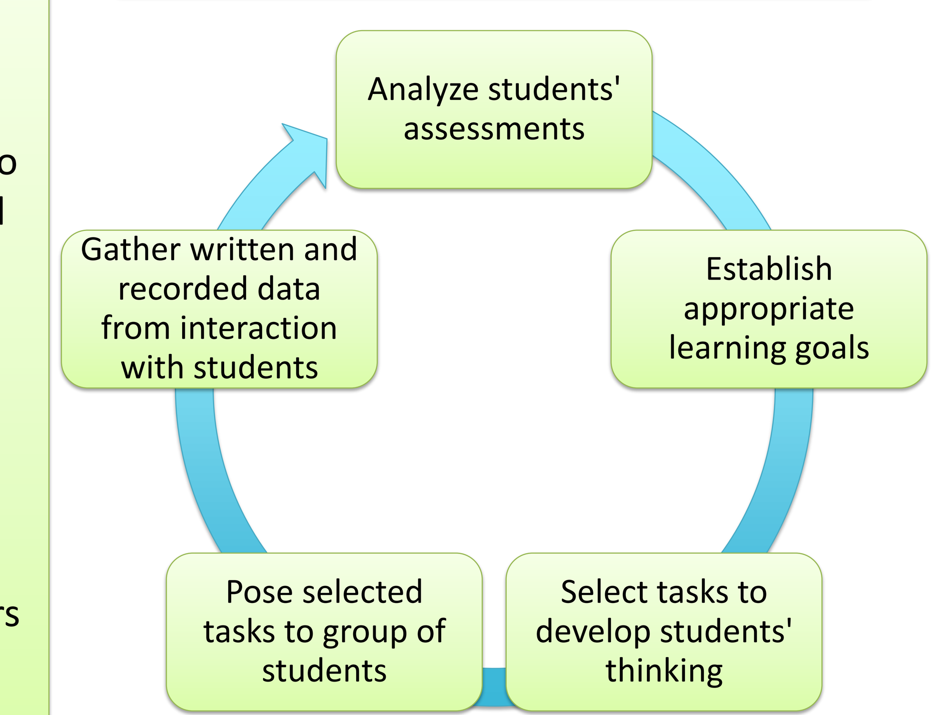
(Illustrative Mathematics b, n.d.)

Key problem two (to the left) allowed students to select their own solution strategies. We used it to assess the extent to which they drew upon visual models and/or procedures when solving problems at the outset of the study.

Procedure

- Teach and audio/video record lesson
- Transcribe audio and video
- Analyze the transcript and make conjectures
- Create lesson draft based on conjectures
- Create formal lesson plan after critique
- Practice lesson with mentors and fellow undergraduate researchers
- Teach lesson to students
- Repeat seven times

Instructional Cycle



References

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Tsankova, J. K. & Pjanic, K. (2010). The area model of multiplication of fractions. *Mathematics Teaching in the Middle School, 15*(5), 281-285.

Webel, C., Krupa, E. & McManus, J. (2016). Using representations of fraction multiplication. *Teaching Children Mathematics, 22*(6), 366-373.

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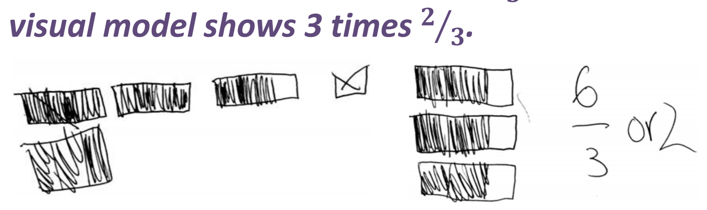
Empirical Teaching and Learning Trajectory:

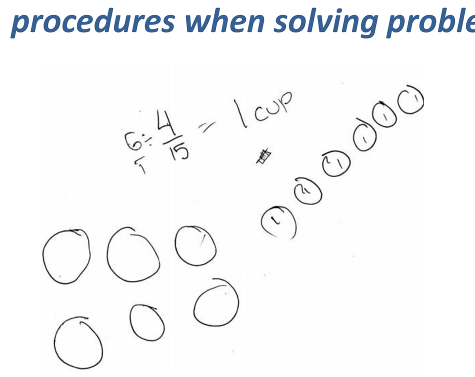
Initial Assessment Results

Most students initially struggled to use visual models to solve problems. They had less trouble with execution of procedures.

Example of student difficulty creating a visual model to represent taking a fractional part of multiple wholes. The student was asked to solve $^{2}/_{3}$ of 3. The student's visual model shows 3 times $^{2}/_{3}$.

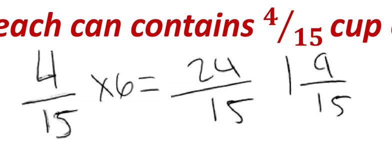


Example of student difficulty in deciding proper procedures when solving problems.



T: So, For a certain brand of orange soda, each can contains 4/15 cup of sugar. Part a is how many cups of sugar are there in six cans of this orange soda.
S1: Hmmm. This is hard.
T: Yeah? What are you thinking about it?
S1: That you have to divide 4/15 into 6 cans.

Example of student strength applying a procedure to find the number of cups of sugar in six cans of soda when each can contains $^{4}/_{15}$ cup of sugar.



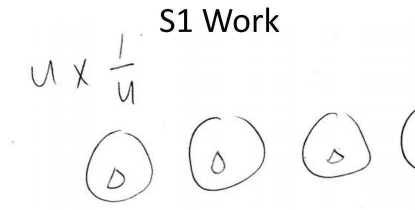
Week 2 Instruction

For this week of instruction we decided to encourage the use of specific language, such as "groups of", along with the use of drawings and pattern blocks. We introduced whole number times fraction number sentences and word problems that involved modeling repeated addition. Students' work on the problems revealed their understanding of language and visual models, as exemplified in the classroom artifacts below.

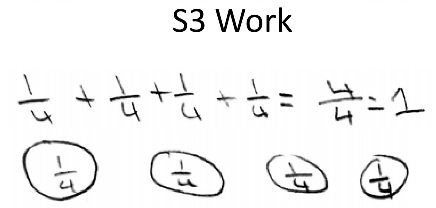
T: Do you know how you can draw a picture of 3 x 5?
S2: Uh, three—uh not really a picture. I was going to say, you can do like fractions with it because 3/15 x 3/15.
T: Okay. What about you -------? Did you have a different answer?
S3: You can do can like circles—three circles with five in each circle.

In the classroom dialogue excerpt to the left, the teacher and students negotiate a possible meaning for 3 x 5 as "three groups of five." Later problems and lessons drew on this conceptualization.

S1 Work



S3 Work

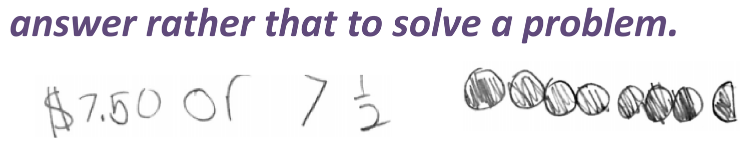


The work to the left provides evidence of students' abilities to think of multiplication of four times one-fourth as repeated addition. We revisited these ideas in later lessons.

Weeks 3-5 Instruction

Students solved and modeled a series of word problems involving multiplication of fractions by whole numbers. Students were hesitant to use conceptual fraction models so we introduced a new modeling technique that forced the students to solve problems without using algorithms.

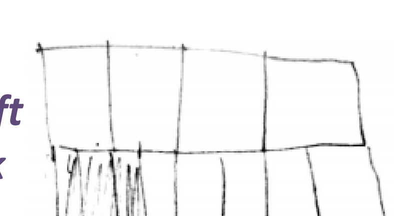
Example of a student using model to just show the answer rather that to solve a problem.



S1: *lays out 5 wholes and divides the rest with a pencil then cuts 3 wholes into fifths*
S2: *stares at all pieces*
S3: *divides every piece with pencil and then cuts every piece into fifths*

Example of students' thinking when they were asked to solve $^{1}/_{5}$ times 8 without using any sort of algorithm. They were provided with eight paper strips and scissors.

The student work to the left provides evidence of a lack of flexible thinking when we tried to return to word problems.



Student model for $^{3}/_{8}$ times 4

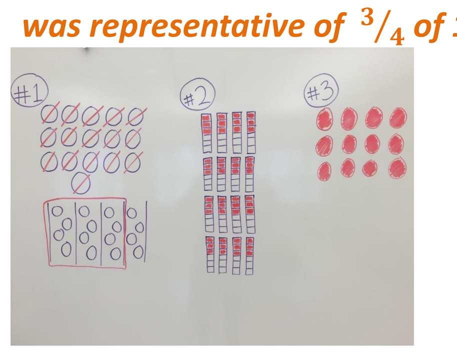
Weeks 6-8 Instruction

Due to students' difficulty constructing models in the previous weeks, we decided to take a different approach that would introduce the students to various models. The instruction during the final three lessons would also give the students multiple opportunities to explain and describe the models in discussion.

Below are the models for $^{2}/_{5}$ of 12 that we used at the beginning of our week 6 lesson. We conjectured that introducing pre-made models would help build students' conceptual understanding. Students were able to compare the models and formulate number sentences to go with them.

T: So what, after discussing these, what can we notice about our models in the middle there? Any similarities, any differences?
S4: They're all representing one equation?
T: Are they?
S4: Yeah.
S3: That the number sentence is 2/5 times 12.

Below is an example of a student's explanation of why a certain model was representative of $^{3}/_{4}$ of 16.

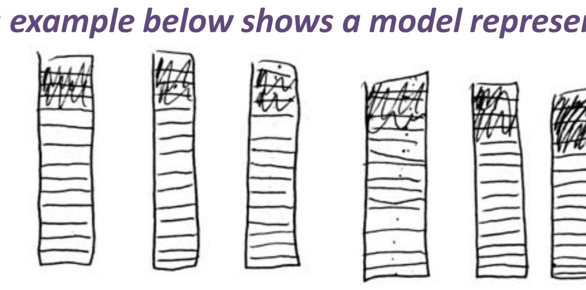


S4: I think that # 1 is correct, cause it that 4 squares *(sic)*. And there's four circles in each square to represent the 16 gems Sarah bought. There is a circle around 3 of the squares to represent that $^{3}/_{4}$ of the gems are diamonds.

Post-Assessment Results

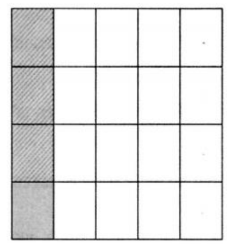
By the end of the seven lessons, students showed improvement with not only creating models to go with number sentences but also creating stories to go with pre-made models.

Student's example below shows a model representing 6 x $^{4}/_{15}$.

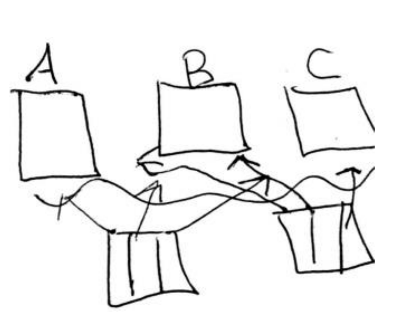


S2: Uh, this person got 4/20 of a car –
T: Of a car?
S2: Mhmm.
T: Okay *chuckles*
S2: Then he multiplied it by 5 and then he got the full car. Because that's only what the insurance claimed.

To the right is the model that was provided in a task which prompted the student to create the story problem shown to the left.



Student's example to the right shows their model when asked to split 2 sandwiches evenly between 3 people.



Reflection and discussion: The standard in our learning progression that was most difficult to obtain was solving real world problems involving fractions and mixed numbers. Students had difficulty creating appropriate models to coincide with the context of the word problems. For example, when asked to show 2/3 of 3, their models initially depicted 3 groups of 2/3. Students seem to benefit from having to explain their thoughts to the class step-by-step. Once they were put in the "teacher's" position they generally realize any mistakes they may have made. Data from our study also point to the conceptual difference between a fraction times a whole number versus a whole number times a fraction. This distinction needs explicit attention. Future research might examine the potential benefits of teaching the case of a fraction times whole number first, in order to examine the impact of such an instructional sequence on students' learning.