

Developing Students' Understanding Of Decimal Numeration In Grade 5

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INTRODUCTION


DECIMAL MISCONCEPTIONS

- ▶ Lengths of decimals do not dictate value
 - ▶ Common strategies do not dictate accuracy
 - ▶ Familiarity with a standard algorithm does not dictate understanding
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- ▶ Conventional teaching strategies often fail to aid with these misconceptions
 - ▶ 1 in 5 pre-service teachers do not have a well-integrated knowledge of decimal numeration
 - ▶ (Stacey 2001)

PURPOSE OF STUDY

- ▶ To examine four students' existing knowledge of decimal numeration and explore strategies for developing their mathematical proficiency in regard to decimals.

RESEARCH QUESTIONS

- ▶ What difficulties do the students have in reasoning about decimals and place value?
 - ▶ What teaching strategies and representations can help the students accurately compare two decimal representations?
 - ▶ What teaching strategies and representations can help the students learn decimal addition and subtraction with understanding?
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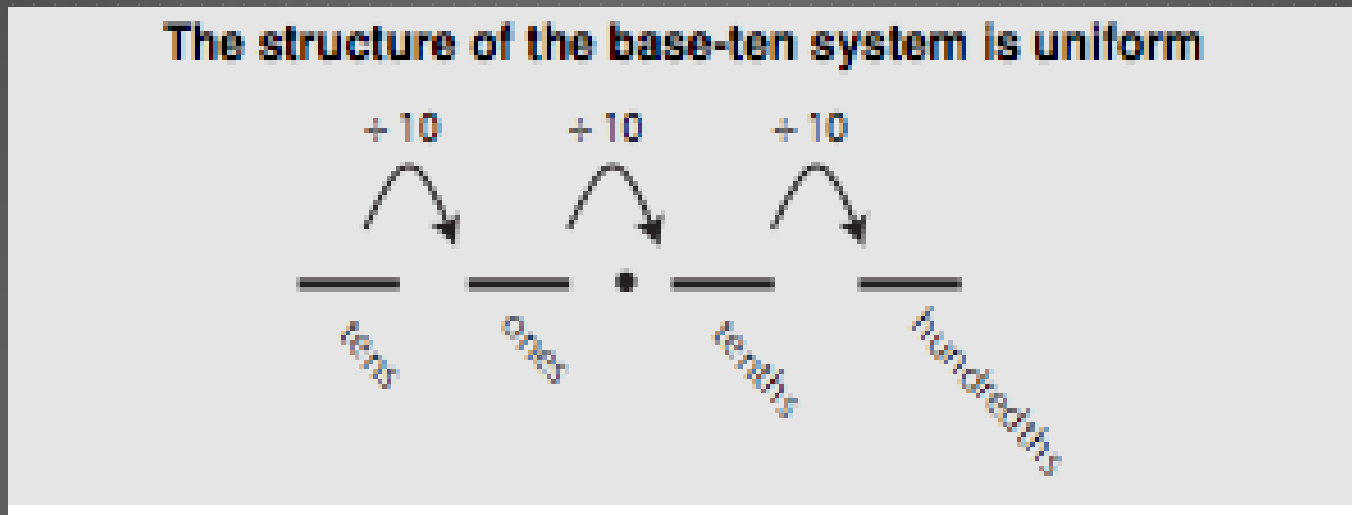
THEORETICAL FRAMEWORK

STRANDS OF MATHEMATICAL PROFICIENCY

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

SUGGESTED LEARNING PROGRESSIONS

- ▶ Generalize place value understanding for multi-digit whole numbers



- ▶ Students extend their understanding of the base-ten system to decimals to the thousandths place

- ▶ Students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers

Subtraction: Decomposing where needed first

decomposing left to right,
1 hundred, then 1 ten

if you subtract the 10

now subtract

if you subtract the 10

$$\begin{array}{r} 425 \\ - 278 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 3 \cancel{2} 15 \\ - 278 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 3 \cancel{2} 15 \\ - 278 \\ \hline 147 \end{array}$$

All necessary decomposing is done first, then the subtractions are carried out. This highlights the two major steps involved and can help to inhibit the common error of subtracting a smaller digit on the top from a larger digit. Decomposing and subtracting can start from the left (as shown) or the right.

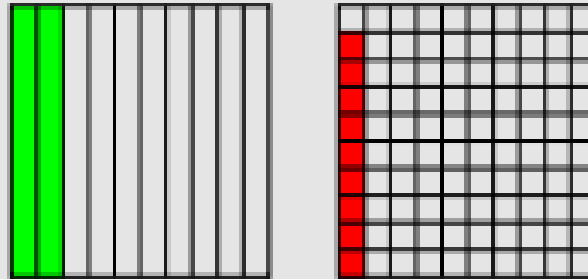
ADDITIONAL LITERATURE

- ▶ **“Promoting Decimal Number Sense and Representational Fluency” (Suh et al., 2008)**

- ▶ **“Investigating Students’ of Decimal Fractions” (Martinie & Bay-Williams, 2003)**

- ▶ **“Progression for the Common Core State Standards for Mathematics” (Common Core Standards Writing Team, 2013)**

Seeing that $0.2 > 0.09$ using a visual fraction model.



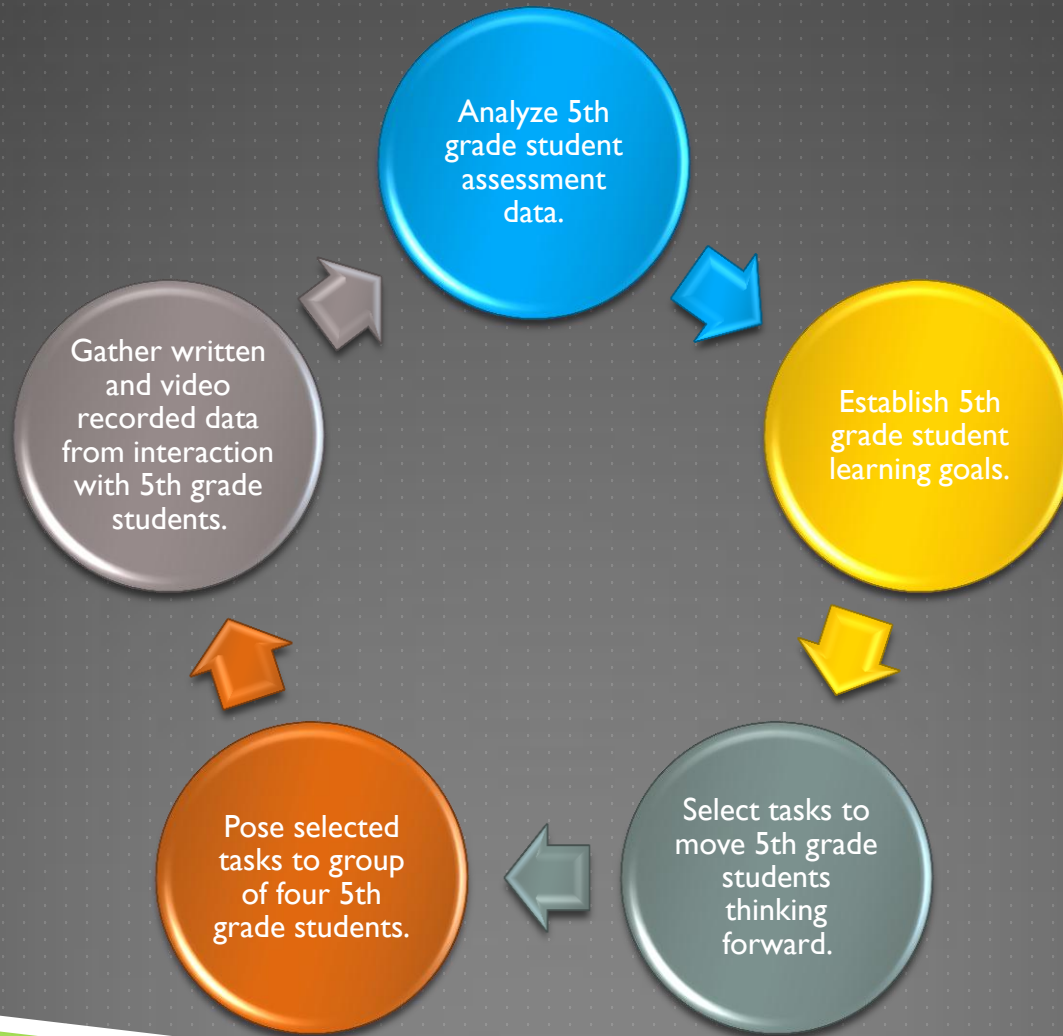
The shaded region on the left shows 0.2 of the unit square, since it is two parts when the square is divided into 10 parts of equal area. The shaded region on the right shows 0.09 of the unit square, since it is 9 parts when the unit is divided into 100 parts of equal area.

METHODOLOGY

PARTICIPANTS AND PROCEDURE

- ▶ Four fifth grade students chosen from local elementary schools
- ▶ Instruction designed to attain to these CCSS:
 - Recognize that a particular digit is ten times smaller than the place on its left.
([CCSS.Math.Content.5.NBT.A.1](#))
 - Compare, read and write decimals to the thousandths place.
([CCSS.Math.Content.5.NBT.A.3](#))
 - Compare, read and write decimals using base-ten numerals, expanded form and number names. ([CCSS.Math.Content.5.NBT.A.3.a](#))
 - Compare decimals using $>$, $=$, $<$ symbols to the thousandths place.
([CCSS.Math.Content.5.NBT.A.3.b](#))
 - Add and subtract decimals of different sizes using drawings and other manipulatives.
([CCSS.Math.Content.5.NBT.B.7](#))

INSTRUCTIONAL CYCLE

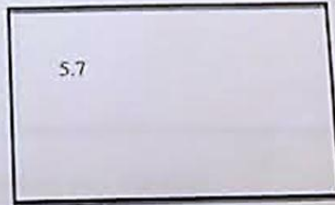
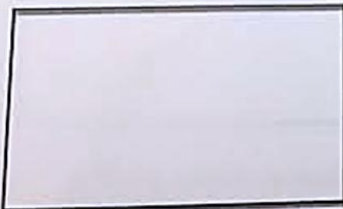


DATA GATHERING AND ANALYSIS

- ▶ Pre/Post Assessment and Interview
- ▶ Sample Questions:

6. Use decimals to solve the following problem. Margie bought 8 apples that cost 47 cents each. She paid with a five-dollar bill. How much change did Margie receive?

8. Write each of this set of numbers in the correct box saying the numbers as you place them in the box.
The box on the left is for numbers smaller than 5.5.
The box on the right is for numbers bigger than 5.5.
The numbers are 5.7 5.35 5.025 5.9 5.24 5.473
The first one has been done for you.



10. Explain the value of the five with the arrow below it. Discuss any mathematical relationships that this number may have with the number on its right and those on its left. Please write the number 55.55 in expanded form.

5 5 • 5 5 5



TUTORING SESSIONS

- ▶ Seven one-hour sessions
- ▶ Video recorded
- ▶ Analyzed based on strands of mathematical proficiency



EMPIRICAL TEACHING AND LEARNING TRAJECTORY

INITIAL ASSESSMENT RESULTS

6. Use decimals to solve the following problem.

Margie bought 8 apples that cost 47 cents each. She paid with a five-dollar bill. How much change did Margie receive?

$$\begin{array}{r} 5 \\ 60 \\ -40 \\ \hline 20 \end{array}$$

8

5.7

6. Use decimals to solve the following problem.

Margie bought 8 apples that cost 47 cents each. She paid with a five-dollar bill. How much change did Margie receive?

$$45.00 - 45.00 =$$
$$\begin{array}{r} 4.70 \\ \hline 9.02 \end{array}$$

6. Use decimals to solve the following problem.

Margie bought 8 apples that cost 47 cents each. She paid with a five-dollar bill. How much change did Margie receive?

$$\begin{array}{r} 0.47 \\ + 0.08 \\ \hline 0.55 \end{array}$$



JD: Reese ran zero point five of a mile. Jen ran zero point forty-five of a mile. Reese thinks she ran more than Jen. Do you agree or disagree? Justify your answer with a written explanation.

LG: ...You can do a visual model.

JD: Ugh

LG: ...Why would you say Reese? Just tell me why you thought that.

JD: Because zero decimal five compared to this,

LG: Is it greater than or less than?

JD: Less than.

LG: Which is less than?

JD: This one

LG: Why? Don't like change your answer just because I'm asking you why. I'm just trying to follow what you're thinking about... What made you say that?...

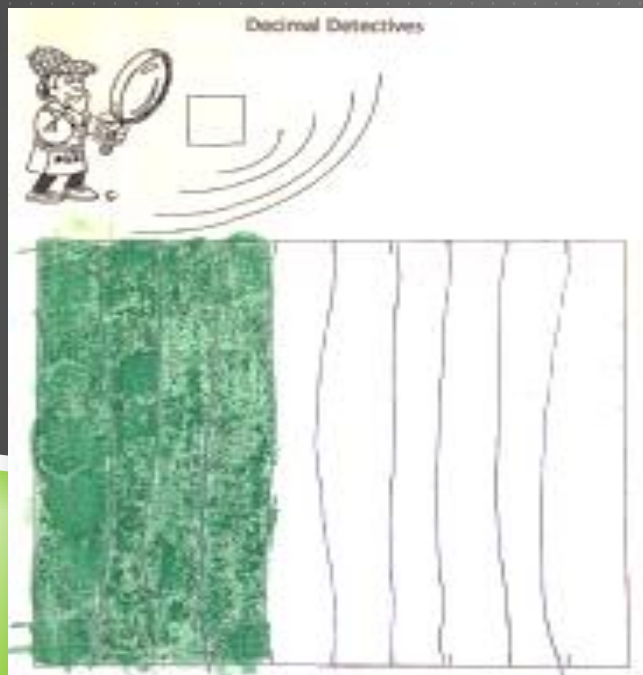
JD: Because if you compare 5 of the zero point five and zero point forty-five, this one would be greater.

LG: But why? Can you draw me a picture?

JD: I don't think I know really how to do this yet.

INSTRUCTIONAL CLUSTER I

- ▶ Understanding place value
- ▶ Using manipulatives
- ▶ Discovering patterns



Kailey: All together it would be ten thousand, but for each box it would be two thousand.

LG: Right, but all together it would be-

Kailey: Ten thousand

LG: So do you think this would be our next piece? What do you think- Which piece does this look like?

Kailey: A rod.

LG: A rod, so do you see any patterns?

Nick: No

Adam: Ooh! I see it I see it!

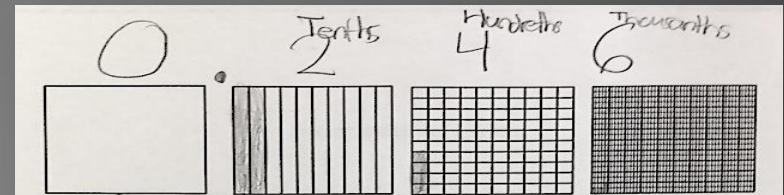
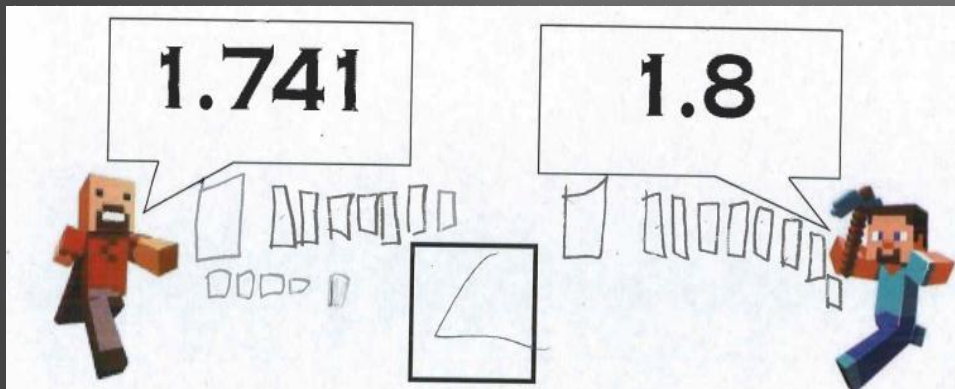
LG: What do you see, Adam?

Adam: 1 times 10 equals 10, 10 times 10 equals 100, 100 times 10 equals 1000 and-

Nick: Ta da!

INSTRUCTIONAL CLUSTER 2

- ▶ Visual Representations of decimals
- ▶ Relationships between decimal places
- ▶ Equivalency
- ▶ Naming Decimals
- ▶ Fluency between representations



INSTRUCTIONAL CLUSTER 3

- ▶ Addition and Subtraction of decimals
- ▶ Concepts of regrouping and borrowing
- ▶ Decimal computation in multiple contexts
- ▶ Multi-step problems

Ahmed has \$1.72 in his piggy bank. While cleaning his room, he found four dollars, two dimes and six pennies. If he puts all of this money in his piggy bank, how much will he have total?

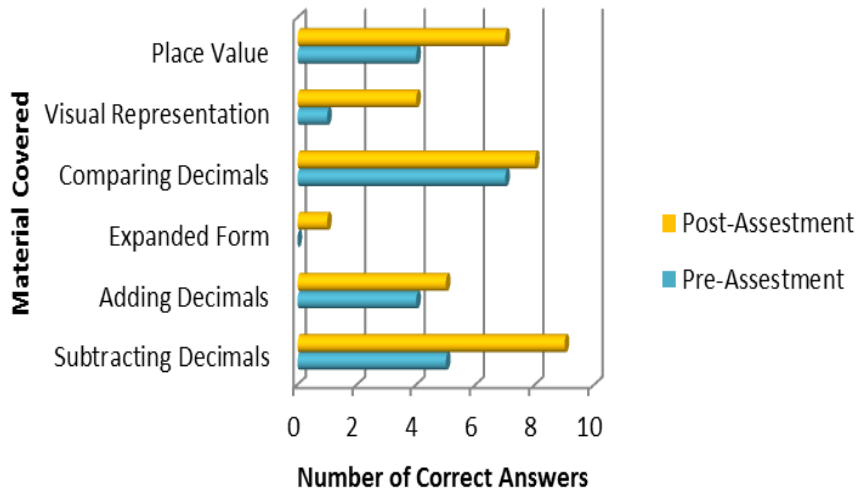


$$\begin{array}{r} 5.98 \\ - 4.26 \\ \hline 1.72 \end{array}$$

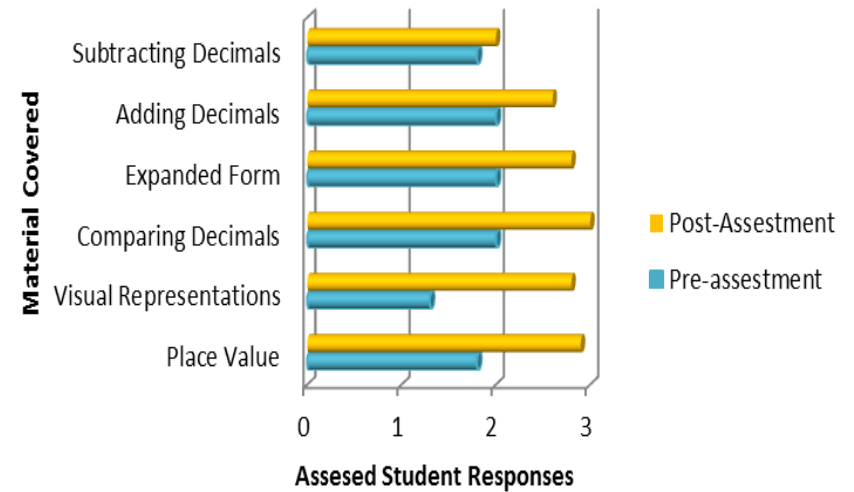


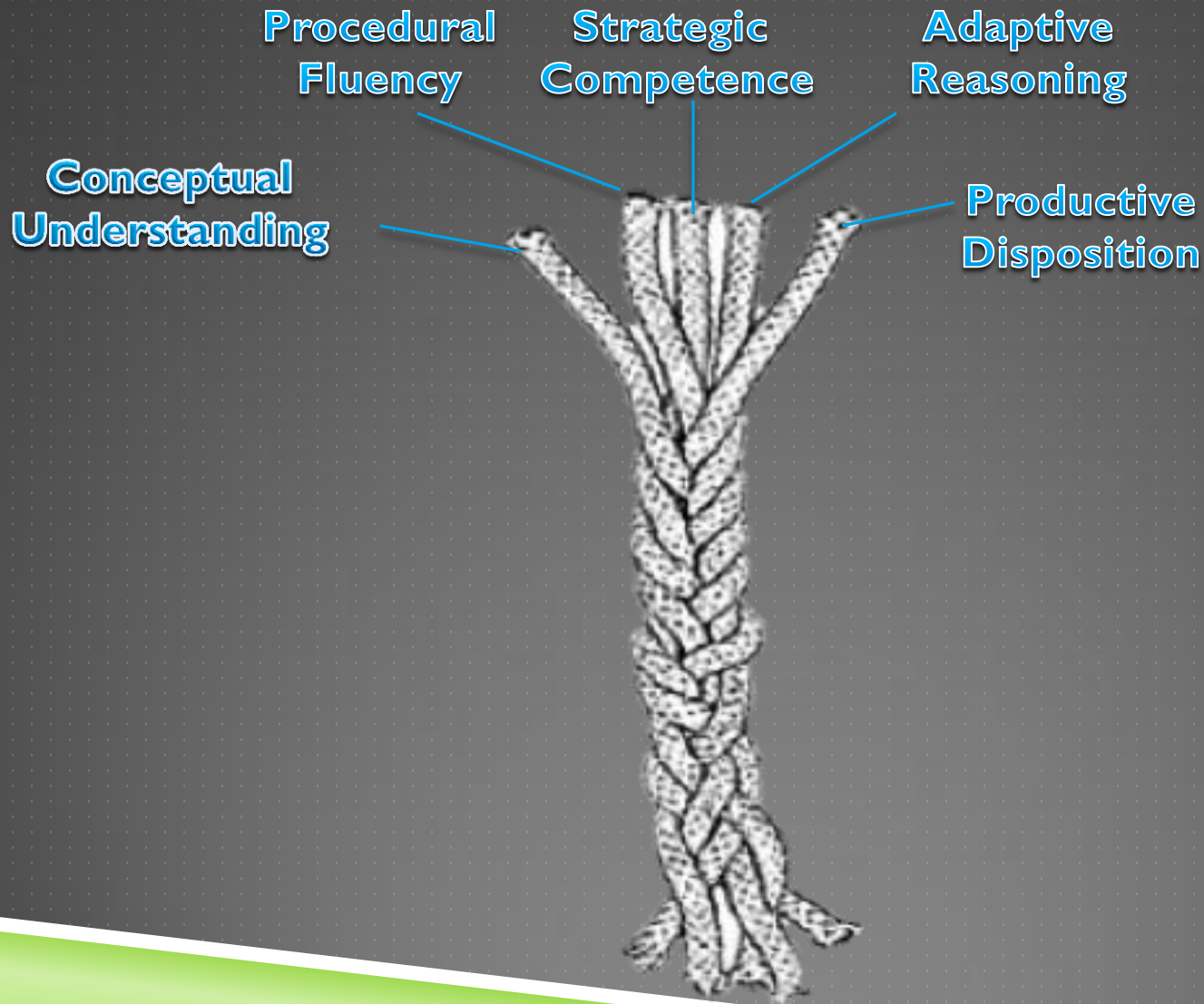
RESULTS

Written Assesment



Student Interview Data





REFLECTION AND DISCUSSION

DIFFICULTIES

▶ CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.



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ADVICE TO TEACHERS

- ▶ Avoid traditional algorithm
- ▶ Develop personal conceptual understanding
- ▶ Employ multiple methods
- ▶ See development as worthy time investment
- ▶ Connect decimals to other mathematical content



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